Answer on Question #59929-Physics-Quantum Mechanics

Q. A particle is confined between rigid walls by a distance L.

(a) Show that the probability P that it will be found within a distance L/3 from one wall is given by

$$P = \frac{1}{3}\left[1 - \frac{sin\frac{2n\pi}{3}}{\frac{2n\pi}{3}}\right]$$

(b) Evaluate probability for (i) n=1, (ii) n=2, (iii) n=3

Solution

(a) A wave function is

$$\psi_n(x) = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$$

The probability P that it will be found within a distance L/3 from one wall is

$$P = \int_0^{\frac{L}{3}} |\psi_n|^2 dx = \int_0^{\frac{L}{3}} \frac{2}{L}\sin^2\frac{n\pi x}{L}dx = \frac{2}{L}\left(\int_0^{\frac{L}{3}}\left(\frac{1}{2} - \frac{1}{2}\cos\frac{2n\pi x}{L}\right)dx\right).$$

$$P = \left(\frac{x}{L} - \frac{\sin\frac{2n\pi x}{L}}{2n\pi}\right)_0^{\frac{L}{3}} = \frac{1}{3}\left[1 - \frac{sin\frac{2n\pi}{3}}{\frac{2n\pi}{3}}\right]$$

(b)(i)

$$P(n=1) = \frac{1}{3}\left[1 - \frac{sin\frac{2\pi}{3}}{\frac{2\pi}{3}}\right] = \frac{1}{3}\left[1 - \frac{\frac{\sqrt{3}}{2}}{\frac{2\pi}{3}}\right] = \frac{1}{3}\left[1 - \frac{3\sqrt{3}}{4\pi}\right] = 0.196$$

(ii)

$$P(n=2) = \frac{1}{3}\left[1 - \frac{sin\frac{4\pi}{3}}{\frac{4\pi}{3}}\right] = \frac{1}{3}\left[1 + \frac{\frac{\sqrt{3}}{2}}{\frac{4\pi}{3}}\right] = \frac{1}{3}\left[1 + \frac{3\sqrt{3}}{8\pi}\right] = 0.402$$

(iii)

$$P(n=3) = \frac{1}{3}\left[1 - \frac{sin\,2\pi}{2\pi}\right] = \frac{1}{3}[1-0] = \frac{1}{3} = 0.333$$