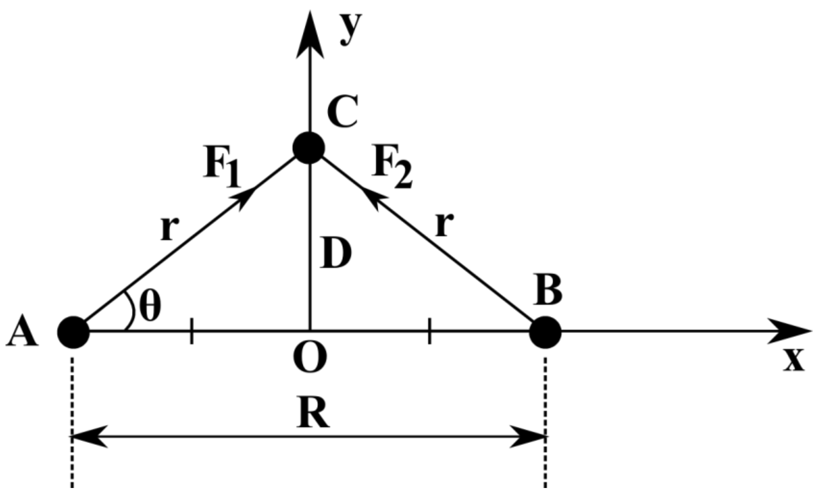
Answer on Question 59862, Physics, Electromagnetism

Question:

Two equal point charges A and B are R distance apart. A third point charge placed on the perpendicular bisector at a distance D from the centre will experience maximum electrostatic force when D equals to what?

Solution:

Here's the sketch of our task:



Two equal point charges A and B are R distance apart. A third point charge C placed on the perpendicular bisector at a distance D from the centre (point O).

Let's first find the net electrostatic force on point charge q_C due to point charges q_A and q_B. It is obviously, that the net electrostatic force on q_C is the vector sum of F_1 and F_2, the forces due to charges q_A and q_B. We can find the magnitudes of these forces from the Coulomb's law:

$$F_1 = k\frac{q_A q_C}{r_{AC}^2},$$

$$F_2 = k\frac{q_B q_C}{r_{BC}^2},$$

here, k is the Coulomb's constant; q_A, q_B, q_C are the point charges; r_{AC}, r_{BC} is the distances between the point charges q_A, q_C and q_B, q_C, respectively.

We can find the distance r_{AC} between the two point charges q_A and q_C from the geometry. By the definition, the perpendicular bisector cuts the line AB into two equal parts AO and OB. Then, from the right triangle AOC we can find r. By the Pythagorean theorem we have:

$$AC^2 = AO^2 + OC^2,$$

$$r_{AC}^2 = \left(\frac{R}{2}\right)^2 + D^2,$$

$$r_{AC} = \sqrt{\left(\frac{R}{2}\right)^2 + D^2}.$$

Similarly, from the triangle BOC we can find the distance r_{BC} between the two point charges q_B and q_C:

$$r_{BC} = \sqrt{\left(\frac{R}{2}\right)^2 + D^2}.$$

Thus, the magnitudes of the forces F_1 and F_2 will be ($q_A = q_B = q$):

$$F_1 = F_2 = k\frac{qq_C}{\left(\frac{R}{2}\right)^2 + D^2}.$$

Then, we can find the projections of the forces F_1 and F_2 on axis x and y:

$$F_x = F_{1x} - F_{2x} = F_1 cos\theta - F_2 cos\theta = 0,$$

$$F_y = F_{1y} + F_{2y} = F_1 sin\theta + F_2 sin\theta = F_1\frac{D}{r_{AC}} + F_2\frac{D}{r_{BC}} =$$

$$= 2k\frac{qq_C}{\left(\left(\frac{R}{2}\right)^2 + D^2\right)} \cdot \frac{D}{\sqrt{\left(\frac{R}{2}\right)^2 + D^2}}.$$

Finally, the net electrostatic force will be:

$$F_{net} = \sqrt{F_x^2 + F_y^2} = 2k\frac{qq_C}{\left(\left(\frac{R}{2}\right)^2 + D^2\right)} \cdot \frac{D}{\sqrt{\left(\frac{R}{2}\right)^2 + D^2}}.$$

For the maximum electrostatic force, we have:

$$\frac{dF(D)}{dD} = 0,$$

$$2kqq_C \cdot \frac{d}{dD}\left(\frac{D}{\left(\left(\frac{R}{2}\right)^2 + D^2\right)^{\frac{3}{2}}}\right) = 0,$$

$$-2kqq_C \cdot \left(\frac{8D^2 - R^2}{4\left(\frac{R^2}{4} + D^2\right)^{\frac{5}{2}}}\right) = 0,$$

$$8D^2 - R^2 = 0,$$

$$8D^2 = R^2,$$

$$D^2 = \frac{R^2}{8},$$

$$D = \frac{R}{2\sqrt{2}}.$$

Answer:

$$D = \frac{R}{2\sqrt{2}}.$$