

Answer on Question #59743 – Physics – Electromagnetism

A dielectric of relative permittivity or dielectric constant 3.0 is filled into the space between the plates of a capacitor. Find the factor by which the capacitance is increased, if the dielectric is only sufficient to fill up 3/4 of the gap.

Solution

Capacitance of plane capacitor is

$$C = \epsilon_0 \epsilon \frac{A}{d}$$

where

ϵ_0 - electric constant;

ϵ - dielectric constant of the material;

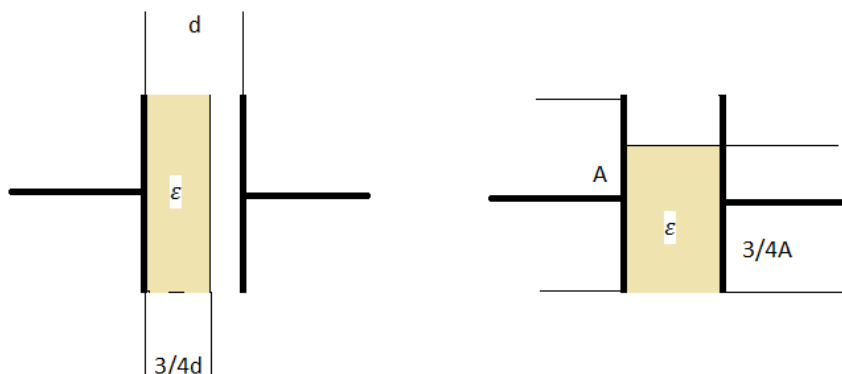
A – area of plate;

d – distance between plates.

Capacitance of empty capacitor:

$$C = \epsilon_0 \frac{A}{d}$$

There are two possibilities to fill the gap:



In the first case we can consider the capacitor to be series connection of two capacitors.

$$\frac{1}{C^*} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_1 = \varepsilon_0 \varepsilon \frac{A}{\frac{3}{4}d} = \varepsilon_0 \varepsilon \frac{4A}{3d}$$

$$C_2 = \varepsilon_0 \frac{A}{\frac{1}{4}d} = \varepsilon_0 \frac{4A}{d}$$

$$C^* = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_0 \varepsilon \frac{4A}{3d} * \varepsilon_0 \frac{4A}{d}}{\varepsilon_0 \varepsilon \frac{4A}{3d} + \varepsilon_0 \frac{4A}{d}} = \frac{\varepsilon_0 A}{d} * \frac{4\varepsilon}{\varepsilon + 3} = C \frac{4\varepsilon}{\varepsilon + 3} = C \frac{4 * 3}{3 + 3} = 2C$$

So we see that in this case the capacitance is increased by 2.

In the other case we can consider the capacitor to be parallel connection of two capacitors.

$$C^{**} = C_1 + C_2$$

$$C_1 = \varepsilon_0 \varepsilon \frac{\frac{3}{4}A}{d} = \varepsilon_0 \varepsilon \frac{3A}{4d}$$

$$C_2 = \varepsilon_0 \frac{\frac{1}{4}A}{d} = \varepsilon_0 \frac{A}{4d}$$

$$C^{**} = \varepsilon_0 \varepsilon \frac{3A}{4d} + \varepsilon_0 \frac{A}{4d} = \frac{\varepsilon_0 A (3\varepsilon + 1)}{4d} = \frac{\varepsilon_0 A}{d} * \frac{3\varepsilon + 1}{4} = C \frac{3 * 3 + 1}{4} = 2,5C$$

So we see that in this case the capacitance is increased by 2,5.

Answer 2 or 2.5