Answer on Question 59468, Physics, Electric Circuits

Question:

A certain generator consists of a rectangular coil of 250 turns and an area of $50\ cm^2$. The coil rotates at a speed of 100 revolutions per second in a horizontal magnetic field $0.3\ T$. Calculate the maximum induced emf in the coil and the induced emf when the plane of the coil is inclined at an angle of 35° to the horizontal:

A) $\mathcal{E}_{max} = 235.6\ V$ and $\mathcal{E} = 193.0\ V$

B) $\mathcal{E}_{max} = 344.2\ V$ and $\mathcal{E} = 230.2\ V$

C) $\mathcal{E}_{max} = 144.3\ V$ and $\mathcal{E} = 96.5\ V$

D) $\mathcal{E}_{max} = 56.4\ V$ and $\mathcal{E} = 26.7\ V$

Solution:

a) Let's use the Faraday's law and find the emf generated between the ends of the coil:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(NBAcos\theta)}{dt},$$

here, $\mathcal{E}$ is the emf generated between the ends of the coil, Φ_B is the magnetic flux through the coil, N is the number of turns of the coil, B is the magnetic field, A is the cross-sectional area of the coil, θ is the angle between the magnetic field and the normal to the plane of the coil.

Since $\theta = \omega t$, we get:

$$\mathcal{E} = -\frac{d(NBAcos\omega t)}{dt} = -NBA\frac{d(cos\omega t)}{dt} = NBA\omega sin\omega t.$$

The maximum value of the emf induced in the coil when $\theta = \omega t = 90^{\circ}$, so that the coil is in the plane of the magnetic field:

$$\mathcal{E}_{max} = NBA\omega,$$

here, ω is the angular frequency with which the coil rotates in a magnetic field.

Let's convert rev/s to rad/s:

$$\omega = \left(100\ \frac{rev}{s}\right) \cdot \left(2\pi \frac{rad}{1\ rev}\right) = 628.32\ \frac{rad}{s}.$$

Finally, substituting ω into the formula for $\mathcal{E}_{max}$, we can calculate the maximum value of the induced emf:

$$\mathcal{E}_{max} = NBA\omega = 250\ turns \cdot 0.3\ T \cdot 5 \cdot 10^{-3}\ m^2 \cdot 628.32\ \frac{rad}{s} = 235.6\ V.$$

b) To find the induced emf when the plane of the coil is inclined at an angle of $\theta = 35^{\circ}$ to the horizontal (to the lines of the magnetic field), we can use the formula:

$$\mathcal{E} = NBA\omega sin\omega t.$$

Let's substitute the numbers:

$$\mathcal{E} = NBA\omega sin\omega t = 250\ turns \cdot 0.3\ T \cdot 5 \cdot 10^{-3}\ m^2 \cdot 628.32\ \frac{rad}{s} \cdot sin35^{\circ} =$$
$$= 135.13\ V.$$

Answer:

$\mathcal{E}_{max} = 235.6\ V,\ \ \mathcal{E} = 135.13\ V.$