A projectile shot at an angle of 60 degrees above the horizontal strikes a wall 25 m away at a point 16 m above the level of projection. What is the magnitude of the velocity with which the projectile hits the wall?

a) 25.44 m/s

b) 21.11 m/s

c) 11.62 m/s

d) 14.63 m/s

Solution

The equations of motion of projectile are:

$$x = v_0 \cos \theta \, t$$

$$y = v_0 \sin \theta \, t - \frac{gt^2}{2}$$

The components of the velocity with which the projectile hits the wall are

$$v_x = v_0 \cos \theta$$

$$v_y = v_0 \sin \theta - gt.$$

$$t = \frac{x}{v_0 \cos \theta}.$$

$$y = v_0 \sin \theta \left(\frac{x}{v_0 \cos \theta} \right) - \frac{g \left(\frac{x}{v_0 \cos \theta} \right)^2}{2}$$

$$y = x \tan \theta - \frac{gx^2}{2v_0^2 \cos^2 \theta}$$

$$v_0 = \frac{x}{\cos \theta} \sqrt{\frac{g}{2(x \tan \theta - y)}} = \frac{25}{\cos 60} \sqrt{\frac{9.8}{2(25 \tan 60 - 16)}} = 21.1825 \frac{m}{s}.$$

$$t = \frac{x}{v_0 \cos \theta} = \frac{25}{21.1825 \cos 60} = 2.36044 \, s.$$

$$v_x = v_0 \cos \theta = 21.1825 \cos 60 = 10.59125 \frac{m}{s}.$$

$$v_y = v_0 \sin \theta - gt = 21.1825 \sin 60 - 9.8 \cdot 2.36044 = -4.78773 \frac{m}{s}.$$

The magnitude of the velocity with which the projectile hits the wall is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10.59125)^2 + (-4.78773)^2} = 11.62 \frac{m}{s}.$$

Answer: c) 11.62 m/s.