Answer on Question 59308, Physics, Mechanics, Relativity

Question:

A pipe contains a gradually tapering section where the diameter decreases from $400\,mm$ to $250\,mm$. The pipe contains an incompressible fluid of density $1000\,kgm^{-3}$ and runs full. If the flow velocity is $2\,ms^{-1}$ in the smaller diameter, determine:

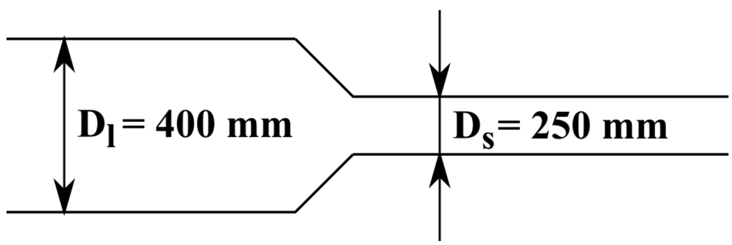
a) the velocity in the larger diameter

b) the volume flow rate

c) the mass flow rate

Solution:

Here's the sketch of our task:



a) We can find the velocity of the fluid from the Law of Continuity:

$$A_l v_l = A_s v_s,$$

here, $A_l = \pi r_l^2$, $A_s = \pi r_s^2$ are the large and small cross-sectional areas of the pipe, respectively; r_l, r_s are the large and small radii of the pipe, respectively; v_l is the velocity of the fluid in the larger diameter, v_s is the the velocity of the fluid in the smaller diameter.

Then, from this formula we can calculate the velocity of the fluid in the larger diameter:

$$v_l = \frac{A_s v_s}{A_l} = v_s \cdot \frac{\pi r_s^2}{\pi r_l^2} = 2\,ms^{-1} \cdot \frac{\pi \cdot (125 \cdot 10^{-3}\,m)^2}{\pi \cdot (200 \cdot 10^{-3}\,m)^2} = 0.78\,ms^{-1}.$$

b) Let us determine the volume flow rate – the rate of flow through the volume V per unit time t:

$$V = vtA,$$

$$\frac{\Delta V}{\Delta t} = A_l v_l = \frac{\pi D_l^2}{4} v_l = \frac{3.14}{4}(400 \cdot 10^{-3}\ m)^2 \cdot 0.78\ \frac{m}{s} = 0.098\ \frac{m^3}{s}.$$

c) As we know the volume flow rate, we can calculate the mass flow rate:

$$\frac{m}{\rho} = vts,$$

$$\frac{\Delta m}{\Delta t} = \rho \frac{\Delta V}{\Delta t} = \rho v_l A_l = 1000\ \frac{kg}{m^3} \cdot 0.098\ \frac{m^3}{s} = 98\ \frac{kg}{s}.$$

Answer:

a) $v_l = 0.78\, ms^{-1}$.

b) $\frac{\Delta V}{\Delta t} = 0.098\ \frac{m^3}{s}$.

c) $\frac{\Delta m}{\Delta t} = 98\ \frac{kg}{s}$.