Answer on Question # 59213 – Physics – Mechanics | Relativity

The power (in watts) from an engine is given by the equation $P = (80t)^{1.3} + 5t$ where t is the time in seconds. Draw a graph of power against time for the engine. From the graph approximate the energy produced between 3 and 7 seconds. Using an appropriate method of integration to calculate the energy produced, and compare the answer with your approximation.

Solution:

The graph of power against time (Figure 1):

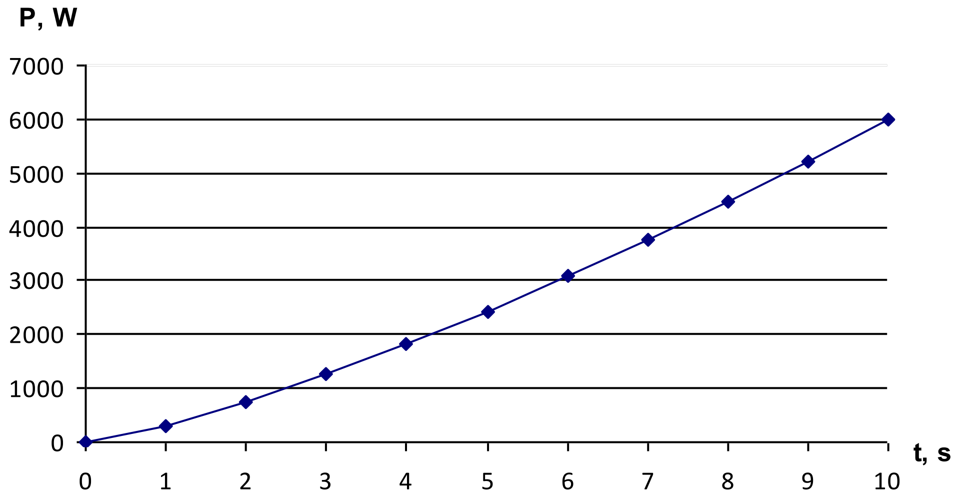

Figure 1 – Power against time

The energy produced between 3 and 7 seconds is numerically equal to the area under the graph, enclosed by the graph, the x axis and vertical lines t = 3 s and t = 7 s. It can be approximately calculated as the total area of one rectangle and one triangle:

$$E_{area} = S\left[(3;0),(3;P(3)),(7;P(7)),(7;0)\right] =$$

$$\approx S_{rect}\left[(3;0),(3;P(3)),(7;P(3)),(7;0)\right] + S_{tri}\left[(3;P(3)),(7;P(7)),(7;P(3))\right] =$$

$$= (7-3)\times(P(3)-0) + \frac{(7-3)\times(P(7)-P(3))}{2} =$$

$$= (7-3)\times(P(3)-0) + \frac{(7-3)\times(P(7)-P(3))}{2} =$$

$$= (7-3)\times((80\times3)^{1,3} + 5\times3) + \frac{(7-3)\times(\left[(80\times7)^{1,3} + 5\times7\right] - \left[(80\times3)^{1,3} + 5\times3\right])}{2} =$$

$$=(7-3)\times((80\times3)^{1.3}+5\times3)+\frac{(7-3)\times(\left[(80\times7)^{1.3}+5\times7\right]-\left[(80\times3)^{1.3}+5\times3\right])}{2}=10060.96\left[\mathrm{J}\right].$$

The energy can also be calculated by integrating the equation of power over the time:

$$E_{int}=\int_3^7\left((80t)^{1.3}+5t\right)dt=\left(80^{1.3}\frac{t^{2.3}}{2.3}+5\frac{t^2}{2}\right)\Big|_3^7=$$

$$=\left(80^{1.3}\times\frac{7^{2.3}}{2.3}+5\times\frac{7^2}{2}\right)-\left(80^{1.3}\times\frac{3^{2.3}}{2.3}+5\times\frac{3^2}{2}\right)=11499.16-1643.07=9856.09\left[\mathrm{J}\right].$$

The relative deviation of the approximate value of energy from the exact value:

$$\delta=\frac{\left|E_{int}-E_{area}\right|}{E_{int}}\times100\%=\frac{\left|9856.09-10060.96\right|}{9856.09}\times100\%=2.08\left[\%\right].$$

Since the graph of power against time is close to linear, the relative deviation of the approximate value of energy from the exact value is small.

Answer: $E_{area}\approx10060.96\left[\mathrm{J}\right];\ E_{int}=9856.09\left[\mathrm{J}\right];\ \delta=2.08\left[\%\right].$