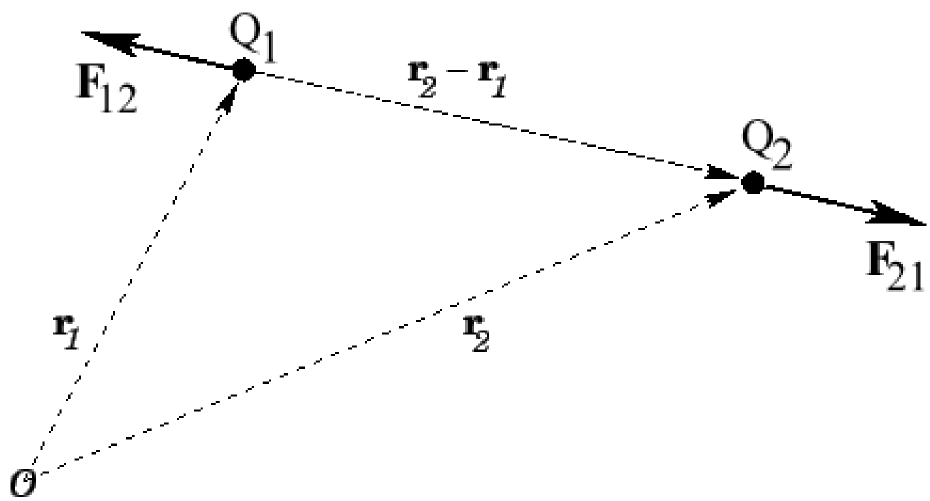
Answer on Question 58944, Physics, Electromagnetism

Question:

Two charges $Q_1 = 500\,\mu C$ and $Q_2 = 100\,\mu C$ are located on the XY plane at the positions $\overrightarrow{r_1} = 3\vec{j}\,m$ and $\overrightarrow{r_2} = 4\vec{i}\,m$. Find the force exerted on the Q_2.

Solution:



We can find the force exerted on the charge Q_2 from the Coulomb's law. Coulomb's law states that the force of attraction or repulsion between two electrically charged particles is directly proportional to the magnitude of their charges and inversely proportional to the square of the distance between them. Let's write the Coulomb's law in vector notation:

$$\overrightarrow{F_{21}} = k\frac{Q_1 Q_2}{r_{21}^2}\overrightarrow{r_{21}},$$

here, $\overrightarrow{F_{21}}$ is the force exerted on the charge Q_2 due to charge Q_1, $k = 9 \cdot 10^9\, N\frac{m^2}{C^2}$ is the Coulomb's constant, Q_1, Q_2 is the charges, $\overrightarrow{r_{21}}$ is the unit vector, r_{12} is the distance between two charges.

We defining the unit vector as follows:

$$\overrightarrow{r_{21}} = \frac{\overrightarrow{r_{21}}}{|\overrightarrow{r_{21}}|} = \frac{\overrightarrow{r_{21}}}{r_{21}},$$

here, $|\overrightarrow{r_{21}}|$ is the magnitude of the $\overrightarrow{r_{21}}$.

So, we can rewrite our formula:

$$\overrightarrow{F_{21}} = k\frac{Q_1Q_2}{r_{21}^2}\frac{\overrightarrow{r_{21}}}{r_{21}},$$

here, $\overrightarrow{r_{21}} = \overrightarrow{r_2} - \overrightarrow{r_1} = 4\vec{\imath} - 3\vec{\jmath}$ is the vectorial distance between two charges.

We can find the distance between two charges from the Pythagorean theorem:

$$r_{21} = \sqrt{r_1^2 + r_2^2} = \sqrt{(3\ m)^2 + (4\ m)^2} = 5\ m.$$

Substituting into the previous equation r_{21} we get:

$$\overrightarrow{F_{21}} = k\frac{Q_1Q_2}{r_{21}^2}\frac{\overrightarrow{r_{21}}}{r_{21}} = 9\cdot 10^9\ N\frac{m^2}{C^2}\cdot\frac{500\cdot 10^{-6}\ C\cdot 100\cdot 10^{-6} C}{(5\ m)^3}\cdot(4\vec{\imath} - 3\vec{\jmath}) =$$
$$= 3.6\cdot(4\vec{\imath} - 3\vec{\jmath}) = (14.4\vec{\imath} - 10.8\vec{\jmath})\ N.$$

Also, we can find the magnitude of the force from the Pythagorean theorem:

$$F_{21} = \sqrt{F_{21x}^2 + F_{21y}^2} = \sqrt{(14.4\ N)^2 + (-10.8\ N)^2} = 18\ N.$$

Answer:

$\overrightarrow{F_{21}} = (14.4\vec{\imath} - 10.8\vec{\jmath})\ N.$

The magnitude of the force is $F_{21} = 18\ N$.