Answer on Question 58710, Physics, Electric Circuits

Question:

The temperature at which the tungsten filament of a $12\ V$ and $36\ W$ lamp operates is 1730 ℃. If the temperature coefficient of resistance of tungsten is $6 \cdot 10^{-3}\ 1/K$, find the resistance of the lamp at a room temperature of 20 ℃:

a) $10.00\ \Omega$

b) $0.45\ \Omega$

c) $0.39\ \Omega$

d) $4.0\ \Omega$

Solution:

Let's first find the resistance of tungsten filament at temperature 1730 ℃. From the definition of the electrical power we have:

$$P = \frac{V^2}{R}.$$

From this formula we can find the resistance of tungsten filament at temperature 1730 ℃:

$$R = \frac{V^2}{P} = \frac{(12\ V)^2}{36\ W} = 4\ \Omega.$$

First, we can find the resistance of the lamp at a temperature of 0 ℃ from the formula:

$$R_{1730℃} = R_{0℃}[1 + \alpha(T - T_{0℃})],$$

here, $R_{1730℃}$ is the resistance of tungsten filament at temperature 1730 ℃ (or $2003.15\ K$), $R_{0℃}$ is the resistance of tungsten filament at reference temperature 0 ℃ (or $273.15\ K$), $\alpha = 6 \cdot 10^{-3}\ 1/K$ is the temperature coefficient of resistance of tungsten at temperature 0 ℃, T the temperature of the tungsten filament (in our case $2003.15\ K$), $T_{0℃}$ is the reference temperature that α is specified at for the tungsten (in our case $273.15\ K$).

From this formula we can find $R_{0℃}$:

$$R_{0℃} = \frac{R}{[1 + \alpha(T - T_{0℃})]} = \frac{4\ \Omega}{\left[1 + 6 \cdot 10^{-3}\ \frac{1}{K} \cdot (2003.15\ K - 273.15\ K)\right]} = 0.35\ \Omega.$$

Then, we can find the resistance of the lamp at a temperature of 20 ℃ just using the same formula:

$$R_{20℃} = R_{0℃}[1 + \alpha(T - T_{0℃})] =$$
$$= 0.35\ \Omega \cdot \left[1 + 6 \cdot 10^{-3}\ \frac{1}{K} \cdot (293.15\ K - 273.15\ K)\right] = 0.39\ \Omega.$$

Answer:

c) $R_{20℃} = 0.39\ \Omega.$