

How long will a $5M_{\odot}$ star burn hydrogen as fuel, given that the Sun will do so for about 10^{10} years?

Solution:

Star's lifetime equals to $t = \frac{E_{released}}{L}$, where $E_{released}$ - energy released after all hydrogen fusion into helium, L - star luminosity.

Solar luminosity equals to $3.828 \cdot 10^{33} \text{ erg/s}$. For the star with $M=5M_{\odot}$ the released amount of energy will be proportional to the mass, i.e.

$$5 \cdot E_{released} = 5 \cdot 10^{10} \text{ y} \cdot 31556926 \frac{\text{s}}{\text{y}} \cdot 3.828 \cdot 10^{33} \frac{\text{erg}}{\text{s}} = 5 \cdot 1.21 \cdot 10^{51} \text{ erg}.$$

The increasing of mass leads to the increase of luminosity which is roughly proportional to $\frac{L}{L_{\odot}} = \left(\frac{M}{M_{\odot}}\right)^a$, where $a \approx 3.9$

$$\frac{L}{L_{\odot}} = 5^{3.9} \approx 532.$$

$$\text{Then } L = 532 \cdot L_{\odot} = 532 \cdot 3.828 \cdot 10^{33} \text{ erg/s}$$

The final expression as follows

$$t = \frac{5 \cdot 1.21 \cdot 10^{51} \text{ erg}}{532 \cdot 3.828 \cdot 10^{33} \text{ erg/s}} = 9.4 \cdot 10^7 \text{ y}$$

Answer: the star will burn its fuel in $9.4 \cdot 10^7 \text{ y}$