Question #58685, Physics / Mechanics | Relativity

An object is dropped from a height of 75.0 m above ground level. (a) Determine the distance traveled during the first second. (b) Determine the final velocity at which the object hits the ground. (c) Determine the distance traveled during the last second of motion before hitting the ground.

Solution:

Let hold the y-axis in the direction of gravity.

Initial velocity $v_0 = 0\ m/s$,

Initial height $h_0 = 0\ m$, and finish height $h_f = 75.0\ m$

gravitational acceleration $g = 9.8\ m/s^2$

a) Determine the distance traveled during the first second.

Let S_0 - the distance traveled during the first second.

Motion equation:

$h(t) = h_0 + v_0 t + \frac{gt^2}{2}$, where $v_0 = 0$, then $h(t) = \frac{gt^2}{2}$

$S_0 = h(1) - h(0) = h_0 + \frac{gt^2}{2} - h_0 = \frac{gt^2}{2} = \frac{9.81 \cdot 1^2}{2} = 4.9[m];$

b) Determine the final velocity at which the object hits the ground

let v_f – velocity of object, when it falling to the ground

$$v(t) = v_0 + gt = gt$$

t_f – moment falling to the ground, and obtain from condition: $h_f = h(t) = 75.0m$

$$h(t_f) = \frac{gt^2}{2} = 75.0\ m$$

$$t_f = \sqrt{\frac{2h_f}{g}};$$

Then

$$v_f = gt = g \cdot \sqrt{\frac{2h_f}{g}} = \sqrt{2gh_f} = \sqrt{2 \cdot 9.8 \cdot 75.0} \approx 38{,}3\ [m/s]$$

c) Determine the distance traveled during the last second of motion before hitting the ground.

let S_f - the distance traveled during the last second of motion before hitting the ground, and $t_{f-1} = t_f - \Delta t$ – moment of time for a second before hitting the ground, where $\Delta t = 1s$

t_f known from b): $t_f = \sqrt{\frac{2h_f}{g}}$

Then

$$S_f = h(t_f) - h(t_{f-1}) = \frac{gt_f^2}{2} - \frac{g(t_f - \Delta t)^2}{2} = \frac{g}{2}\left(t_f^2 - (t_f - \Delta t)^2\right)$$
$$= \frac{g}{2}(t_f^2 - t_f^2 + 2t_f \cdot \Delta t - \Delta t^2) =$$

$$= \Delta t \cdot \frac{g}{2}(2t_f - \Delta t) = g \cdot t_f \cdot \Delta t - \frac{g \cdot \Delta t^2}{2} = g \cdot \sqrt{\frac{2h_f}{g}} \cdot \Delta t - \frac{g}{2} \cdot \Delta t^2 = \sqrt{2h_f g} \cdot \Delta t - \frac{g \cdot \Delta t^2}{2} =$$

$$= \sqrt{2 \cdot 75.0 \cdot 9.8} \cdot 1 - \frac{9.8 \cdot 1^2}{2} = 38,3 - 4.9 = 33.4\ [m]$$

Answer:

a) $S_0 = \frac{gt^2}{2} = 4.9[m]$

b)$v_f = \sqrt{2gh_f} \approx 38,3\ [m/s]$

c) $S_f = \sqrt{2h_f g} \cdot \Delta t - \frac{g \cdot \Delta t^2}{2} \approx 33.4\ [m]$