

Answer on Question #58682 – Physics/Mechanics – Relativity

A coin is dropped from a hot-air balloon that is 300 m above the ground and rising at 10.0 m/s upward. For the coin, find (a) the maximum height reached, (b) its position and velocity 4.00 s after being released, and (c) the time before it hits the ground.

Solution:

General formula for position is:

$$x = x_0 + v_0 \cdot t + \frac{a \cdot t^2}{2}$$

Where x_0 is starting distance, v_0 is starting velocity and a is acceleration. For this task, $x_0 = 300 \text{ m}$, $v_0 = 10 \frac{\text{m}}{\text{s}}$ and $a = -g = -9.8 \frac{\text{m}}{\text{s}^2}$, because acceleration is directed downwards.

a) To find maximum height reached, one can derive $x(t)$ by t and get:

$$\frac{d}{dt}x(t) = v_0 + at$$

Extremum of function demands first derivative to be equal to zero, thus:

$$v_0 + at = 0; t = -\frac{v_0}{a} = -\frac{10 \frac{\text{m}}{\text{s}}}{9.8 \frac{\text{m}}{\text{s}^2}} \approx 1.02 \text{ s}$$

Maximum height will be reached at $t = 1.02 \text{ s}$ and will be equal to:

$$x(t = 1.02 \text{ s}) = 300 + 10 \cdot 1.02 - \frac{9.8 \cdot 1.02^2}{2} = 305.102 = 305.1(\text{m})$$

b) Velocity after 4.00 s after release can be calculated as follows:

$$v(t = 4.00 \text{ s}) = v_0 + at = 10 - 9.8 \cdot 4 = -29.2 \left(\frac{\text{m}}{\text{s}}\right)$$

negative sign means coin is travels downwards. Position after 4.00 s after release:

$$x(t = 4.00 \text{ s}) = x_0 + v_0 t + \frac{a \cdot t^2}{2} = 300 + 10 \cdot 4.00 - \frac{9.8 \cdot 4^2}{2} = 261.6 \text{ (m)}$$

above the ground.

c) Coin hitting the ground means it's position is 0 m. To get time, one can solve the equation:

$$0 = x_0 + v_0 t + \frac{a \cdot t^2}{2}$$

$$D = b^2 - 4ac = v_0^2 - 4 \cdot \frac{a \cdot x_0}{2} = v_0^2 - 2ax_0 = 100 - 2 \cdot (-9.8) \cdot 300 = 5980$$

$$t = \frac{-v_0 \pm \sqrt{D}}{a} = \frac{-10 \pm 77.33}{9.8}$$

There are two roots, one of which is negative – but because time can't be negative, that root is not physical one, so it's neglected. The one that's left:

$$t = 6.87 \text{ s}$$

Answer:

a) Maximum height reached $x = 305.1(\text{m})$.

b) Velocity after 4.00 s $v = -29.2 \left(\frac{m}{s}\right)$, position after 4.00 s $x = 261.6 \text{ (m)}$
c) Total time of flight $t = 6.87 \text{ s}$.

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