Answer on Question 58656, Physics, Quantum Mechanics

Question:

A $511\, keV$ gamma-ray photon is Compton-scattered from a free electron in an aluminum block.

a) What is the wavelength of the incident photon?

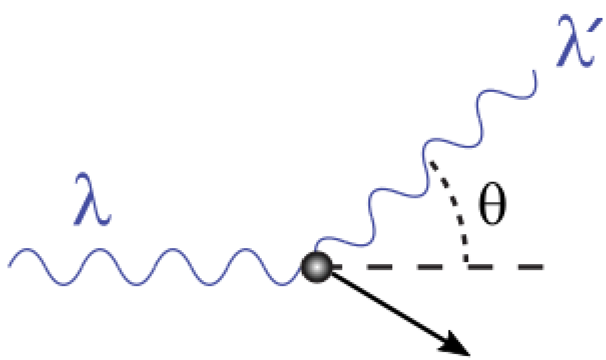
b) What is the wavelength of the scattered photon?

c) What is the energy of the scattered photon?

Assume a scattering angle of 72.0°.

Solution:

In this question, we are dealing with the famous Compton effect. Let's recall what we know about the Compton effect.



A photon of wavelength λ comes in from left, collides with a free electron in an aluminum block, and a new photon of wavelength λ' emerges at an angle θ (in our case, $\theta = 72.0^{\circ}$). Part of the energy of the photon is transferred to the recoiling electron (the arrow in the picture indicates the direction of motion of the electron).

a) First of all, let's find the wavelength of the incident photon. There is an inverse relationship between the energy of the photon and the wavelength of the light given by the equation:

$$E = \frac{hc}{\lambda},$$

here, $h = 4.135 \cdot 10^{-15}\ eV \cdot s$ is the Planck's constant, c is the speed of light, λ is the wavelength of the light.

Then, from this formula we can calculate the wavelength of the incident photon:

$$\lambda = \frac{hc}{E} = \frac{4.135 \cdot 10^{-15}\ eV \cdot s \cdot 3 \cdot 10^{8}\ \frac{m}{s}}{511 \cdot 10^{3}\ eV} = 2.43 \cdot 10^{-12}\ m = 2.43\ pm.$$

b) We can find the wavelength of the scattered photon, λ', from the Compton Scattering equation:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - cos\theta),$$

here, λ is the initial wavelength of the photon, λ' is the wavelength after scattering, $\frac{h}{m_e c}$ is the Compton wavelength, and it is equal to $2.43 \cdot 10^{-12}\ m$, h is the Planck's constant, m_e is the electron rest mass, c is the speed of light, θ is the scattering angle.

Therefore, from this formula we can calculate λ':

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - cos\theta) = 2.43 \cdot 10^{-12}\ m + 2.43 \cdot 10^{-12}\ m \cdot (1 - cos72.0^{\circ}) =$$

$$= 2.43 \cdot 10^{-12}\ m + 1.68 \cdot 10^{-12}\ m = 4.11 \cdot 10^{-12}\ m = 4.11\ pm.$$

c) Finally, using the relationship between the energy of the photon and the wavelength of the light, we can find the energy of the scattered photon:

$$E' = \frac{hc}{\lambda'},$$

here, E' is the energy of the scattered photon, $h = 4.135 \cdot 10^{-15}\ eV \cdot s$ is the Planck's constant, c is the speed of light, λ' is the wavelength of the scattered photon.

Therefore, from the last formula we can calculate the energy of the scattered photon:

$$E' = \frac{hc}{\lambda'} = \frac{4.135 \cdot 10^{-15}\ eV \cdot s \cdot 3 \cdot 10^{8}\ \frac{m}{s}}{4.11 \cdot 10^{-12}\ m} = 301824\ eV \sim 302\ keV.$$

Let's check our calculations by applying the Law of Conservation of Energy:

$$E = E' + KE_e,$$

here, E is the energy of the photon before the collision with a free electron in an aluminum block, E' is the energy of the photon after the collision with the free electron in an aluminum block and KE_e is the kinetic energy imparted to recoiling electron.

Taking into account the inverse relationship between the energy of the photon and the wavelength of the light, we get:

$$\frac{hc}{\lambda} = \frac{hc}{\lambda'} + KE_e.$$

From this formula, we can find the kinetic energy imparted to recoiling electron:

$$KE_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right).$$

Let's substitute the numbers:

$$KE_e = 4.135 \cdot 10^{-15}\ eV \cdot s \cdot 3 \cdot 10^8\ \frac{m}{s} \cdot \left(\frac{1}{2.43 \cdot 10^{-12}\ m} - \frac{1}{4.11 \cdot 10^{-12}\ m}\right) =$$
$$= 209\ eV.$$

Let's verify the Law of Conservation of Energy:

$$E = E' + KE_e,$$

$$511\ keV = 302\ keV + 209\ eV,$$

$$511\ keV = 511\ keV.$$

Therefore, we do all the calculations correctly.

Answer:

a) $\lambda = 2.43 \cdot 10^{-12}\ m = 2.43\ pm.$

b) $\lambda' = 4.11 \cdot 10^{-12}\ m = 4.11\ pm.$

c) $E' = 302\ keV.$