

Answer on Question #58601-Physics-Quantum Mechanics

Q. A 511 keV gamma ray photon is Compton scattered from a free electron in an aluminum block. What is wavelength of incident photon? What is wavelength of scattered photon? What is energy of scattered photon?

Solution

Energy E of a photon having wavelength λ can be written as

$$E = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}.$$

The wavelength of incident photon is

$$\lambda = \frac{1240 \text{ eV} \cdot \text{nm}}{511 \cdot 10^3 \text{ eV}} = 2.43 \text{ pm}.$$

The wavelength of scattered photon is

$$\lambda' = \lambda + \frac{h}{m_e c} (1 - \cos \theta), \quad \frac{h}{m_e c} = 2.43 \text{ pm},$$

where θ is a scattering angle.

$$\lambda' = 2.43 \text{ pm} + 2.43 \text{ pm}(1 - \cos \theta) = 2.43(2 - \cos \theta) \text{ pm}$$

The energy of scattered photon is

$$E = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda'} = \frac{1240 \text{ eV} \cdot \text{nm}}{2.43(2 - \cos \theta) \text{ pm}} = \frac{511}{(2 - \cos \theta)} \text{ keV}.$$

Q. show that $\Delta E/E = hf'/m_e c^2 (1 - \cos \theta)$

Solution

$$\lambda' = \lambda + \frac{h}{m_e c} (1 - \cos \theta)$$

$$E' = \frac{hc}{\lambda'}; \quad E = \frac{hc}{\lambda}.$$

$$\frac{\Delta E}{E} = \frac{\frac{hc}{\lambda'} - \frac{hc}{\lambda}}{\frac{hc}{\lambda}} = \frac{\lambda - \lambda'}{\lambda'} = \frac{-\frac{h}{m_e c} (1 - \cos \theta)}{\frac{c}{f'}} = -\frac{hf'}{m_e c^2} (1 - \cos \theta)$$

$$\left| \frac{\Delta E}{E} \right| = \frac{hf'}{m_e c^2} (1 - \cos \theta)$$