Answer on Question 58600, Physics, Quantum Mechanics

Question:

1. Find frequency, wavelength and momentum of photon whose energy is equal to rest energy of electron.

Solution:

a) By the famous Einstein formula (mass-energy equivalence) we have:

$$E = mc^2.$$

The mass-energy equivalence states that every mass has an energy equivalent and vice versa. If we plug into this formula the electron rest mass that is equal to

$m = 9.11 \cdot 10^{-31}\ kg$ and multiply it by c^2, we get the rest energy of electron that is equal to $8.19 \cdot 10^{-14}\ J$.

From the other hand, we know that the energy of photon is equal to

$$E = \frac{hc}{\lambda} = hf,$$

here, $h = 6.626 \cdot 10^{-34}\ J \cdot s$ is Planck's constant, c is the speed of light.

Since we know from the condition of the question that the energy of photon is equal to rest energy of electron, we can equate both formula and get:

$$mc^2 = hf.$$

From the last formula we can find the frequency of photon whose energy is equal to rest energy of electron:

$$f = \frac{mc^2}{h} = \frac{9.11 \cdot 10^{-31}\ kg \cdot \left(3 \cdot 10^8\ \frac{m}{s}\right)^2}{6.626 \cdot 10^{-34}\ J \cdot s} = 1.24 \cdot 10^{20}\ Hz.$$

b) As we know the frequency of the photon, we can find the wavelength of the photon from the formula:

$$f = \frac{c}{\lambda},$$

here, f is the frequency of the photon, λ is the wavelength of the photon, c is the speed of light.

Then, from this formula we can calculate λ:

$$\lambda = \frac{c}{f} = \frac{3 \cdot 10^8 \ \frac{m}{s}}{1.24 \cdot 10^{20} \ Hz} = 2.42 \cdot 10^{-12} \ m = 2.42 \ pm.$$

c) We can calculate the momentum of the photon using the famous De Broglie wavelength formula:

$$\lambda = \frac{h}{p},$$

here, λ is the wavelength of the photon, h is the Planck's constant, p is the momentum of the photon.

Then, from this formula we can calculate p:

$$p = \frac{h}{\lambda} = \frac{6.626 \cdot 10^{-34} \ J \cdot s}{2.42 \cdot 10^{-12} \ m} = 2.74 \cdot 10^{-22} \ kg \cdot \frac{m}{s}.$$

Answer:

a) $f = 1.24 \cdot 10^{20} \ Hz$,

b) $\lambda = 2.42 \cdot 10^{-12} \ m = 2.42 \ pm$,

c) $p = 2.74 \cdot 10^{-22} \ kg \cdot \frac{m}{s}$.

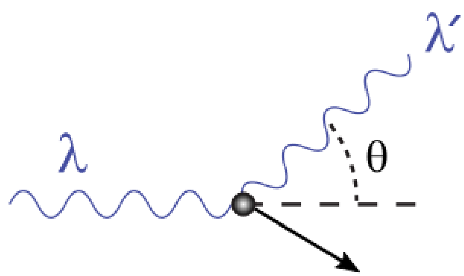
2. Photons of wavelength of $2.17 \ pm$ are incident on free electrons

(a) find wavelength of photon that is scattered at 35° from the incident direction.

(b) do the same if the scattering angle is 115°

Solution:

In this question, we are dealing with the famous Compton effect. Here's the explanation of the Compton effect:



A photon of wavelength λ comes in from left, collides with a free electron at rest, and a new photon of wavelength λ' emerges at an angle θ (in case of (a), $\theta = 35^{\circ}$; in case of (b), $\theta = 115^{\circ}$). Part of the energy of the photon is transferred to the recoiling electron (the arrow in the picture indicates the direction of motion of the electron).

a) We can find λ' from the Compton Scattering equation:

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - cos\theta),$$

here, λ the initial wavelength of the photon, λ' is the wavelength after scattering, $\frac{h}{m_e c}$ is the Compton wavelength, and it is equal to $2.43 \cdot 10^{-12}\ m$, h is the Planck's constant, m_e is the electron rest mass, c is the speed of light, θ is the scattering angle.

Therefore, from this formula we can calculate λ' for the photon that is scattered at 35° from the incident direction:

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - cos\theta) = 2.17 \cdot 10^{-12}\ m + 2.43 \cdot 10^{-12}\ m\ \cdot (1 - cos35^{\circ}) =$$
$$= 2.61 \cdot 10^{-12}\ m = 2.61\ pm.$$

b) We can calculate λ' for the photon that is scattered at 115° from the incident direction from the same formula:

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - cos\theta) = 2.17 \cdot 10^{-12}\ m + 2.43 \cdot 10^{-12}\ m\ \cdot (1 - cos115^{\circ}) =$$
$$= 5.63 \cdot 10^{-12}\ m = 5.63\ pm.$$

Answer:

a) $\lambda'(35^{\circ}) = 2.61 \cdot 10^{-12}\ m = 2.61\ pm$, b) $\lambda'(115^{\circ}) = 5.63 \cdot 10^{-12}\ m = 5.63\ pm$.