

Answer on Question 58586, Physics, Mechanics, Relativity

Question:

Find the kinetic energy of a rigid body rotating about a fixed point.

Solution:

A rigid body rotating with uniform angular speed ω about a fixed point possesses kinetic energy of rotation. We can calculate its value by summing up the individual kinetic energies of all the particles of which the body is composed. A particle of mass m_1 located at distance r_1 from the axis of rotation has kinetic energy given by $\frac{1}{2}m_1v_1^2$, here v_1 is the speed of the particle. Then, we can write the formula for the total kinetic energy:

$$E_k = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \cdots + \frac{1}{2}m_nv_n^2 = \sum_{i=1}^n \frac{1}{2}m_i v_i^2,$$

Each particle of a rigid body rotates with uniform angular speed ω . Then, using the relation between linear and angular variables ($v = \omega r$) and substituting it into the previous equation, we get:

$$E_k = \frac{1}{2}m_1r_1^2\omega^2 + \frac{1}{2}m_2r_2^2\omega^2 + \cdots + \frac{1}{2}m_nr_n^2\omega^2 = \frac{1}{2}\omega^2(m_1r_1^2 + m_2r_2^2 + \cdots m_nr_n^2).$$

Let's denote the factor in parentheses by the letter I (here, I is the moment of inertia of the rotating body with respect to the particular axis of rotation):

$$I = m_1r_1^2 + m_2r_2^2 + \cdots m_nr_n^2 = \sum_{i=1}^n m_i r_i^2.$$

Finally, we can write the kinetic energy of a rigid body rotating about a fixed point as:

$$E_k = \frac{1}{2}I\omega^2.$$

Answer:

$$E_k = \frac{1}{2}I\omega^2.$$