Answer on Question 58542, Physics, Mechanics, Relativity

Question:

A 3.60 kg block is suspended from a spring with $k = 560\ N/m$. A 56.0 g bullet is fired straight up into the block from directly below with a speed of $124\ ms^{-1}$ and becomes embedded in the block.

a) Find the amplitude of the resulting simple harmonic motion.

b) What percentage of the original kinetic energy of the bullet is transferred to mechanical energy of the oscillator?

Solution:

a) Let's first find the final velocity of the combination of bullet and block immediately after the bullet hits the block. Applying the law of conservation of momentum we get:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_f,$$

here, m_1, m_2 is the masses of bullet and block, respectively; v_1, v_2 is the initial velocities of bullet and block, respectively; v_f is the final velocity of the combination of bullet and block immediately after the bullet hits the block.

Since the block is initially at rest, $v_2 = 0\ ms^{-1}$ and we get:

$$m_1 v_1 = (m_1 + m_2) v_f,$$

$$v_f = \frac{m_1 v_1}{(m_1 + m_2)} = \frac{0.056\ kg \cdot 124\ ms^{-1}}{(3.60\ kg + 0.056\ kg)} = 1.9\ ms^{-1}.$$

Then, from the law of conservation of energy, we can find the amplitude of the resulting simple harmonic motion (the potential energy of the spring is equal to the kinetic energy of the combination of block and bullet):

$$PE = KE,$$

$$\frac{1}{2} k A^2 = \frac{1}{2} M_{block+bullet} \cdot v_f^2,$$

$$A = \sqrt{\frac{M_{block+bullet} \cdot v_f^2}{k}} = \sqrt{\frac{3.656\ kg \cdot (1.9\ ms^{-1})^2}{560\ N/m}} = 0.1535\ m = 15.35\ cm.$$

b) Let's first calculate the original kinetic energy of the bullet:

$$KE_{bullet\ original} = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \cdot 0.056\ kg \cdot (124\ ms^{-1})^2 = 430.53\ J.$$

Then, let's calculate the mechanical energy of the oscillator:

$$E_{oscillator} = PE = \frac{1}{2} kA^2 = \frac{1}{2} \cdot 560\ \frac{N}{m} \cdot (0.1535\ m)^2 = 6.597\ J.$$

Finally, we can calculate what percentage of the original kinetic energy of the bullet is transferred to mechanical energy of the oscillator:

$$\frac{E_{oscillator}}{KE_{bullet\ original}} = \frac{6.597\ J}{430.53\ J} \cdot 100\% = 1.53\%.$$

Answer:

a) $A = 0.1535\ m = 15.35\ cm.$

b) $\frac{E_{oscillator}}{KE_{bullet\ original}} = 1.53\%.$