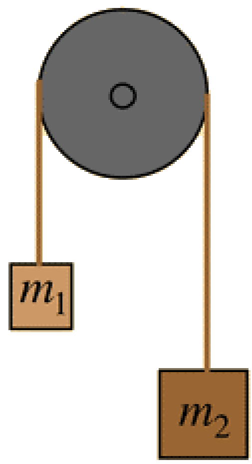
Answer on Question 58534, Physics, Mechanics | Relativity

Question:

How does the force exerted upward by the pulley change as the system begins accelerating?

Answer:

Let’s consider the ideal Atwood’s machine.



a) The ideal Atwood’s machine consists of two objects of mass m_1 and m_2, connected by an inextensible massless string over an ideal massless pulley. Let’s, for example, the mass m_1 be 10 kg and the mass m_2 be 20 kg. Let the acceleration due to gravity g be 9.8 ms^{-2}.

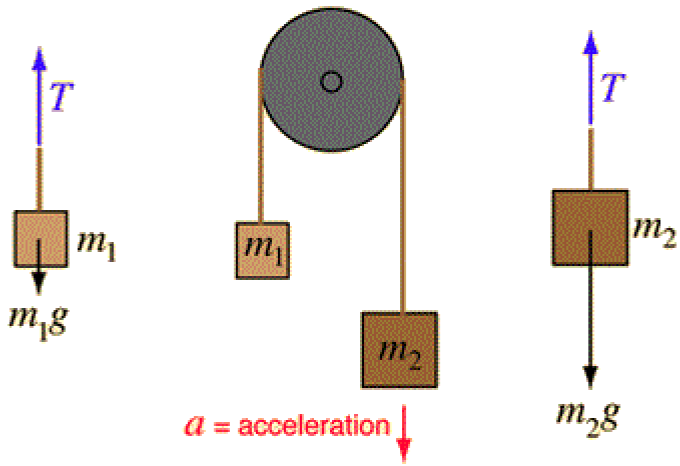
To bring the machine in the position of equilibrium we pull down by hand on the 10 kg mass. In this case, the tension in the string will support the weight of the mass m_2. We can calculate the force of tension from the Newton’s second law of motion:

$$\sum F_y = ma_y,$$

$$T - m_2 g = 0,$$

$$T = m_2 g = 20\ kg \cdot 9.8\ ms^{-2} = 196\ N.$$

Now let assume, that we remove the hand and release the masses m_1 and m_2. We can see, that the mass m_1 accelerates upward and the mass m_2 – downward. First, we need to find the acceleration of the system of masses. In the picture below you can see the free body diagram for this case.



Then, using the Newton's second law of motion we can write the forces affecting m_1 and m_2:

$$T - m_1 g = m_1 a,$$

$$m_2 g - T = m_2 a.$$

Adding these two equations we get:

$$m_2 g - m_1 g = m_1 a + m_2 a,$$

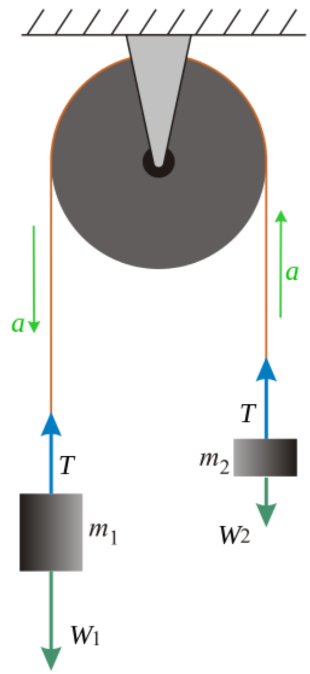
$$a = g \frac{m_2 - m_1}{m_1 + m_2} = 9.8\, ms^{-2} \cdot \frac{20\, kg - 10\, kg}{20\, kg + 10\, kg} = 3.26\, ms^{-2}.$$

Once we know the acceleration of the system, we can calculate the force of tension in the string:

$$T = m_1 a + m_1 g = m_1 (a + g) = 10\, kg \cdot (3.26\, ms^{-2} + 9.8\, ms^{-2}) = 130.6\, N.$$

Therefore, we can conclude that the tension in the string decreases when $m_1 < m_2$ and we release the system of masses.

b) Let's consider the opposite case – the mass m_1 is 20 kg and the mass m_2 is 10 kg. In the picture below you can see the free body diagram for this case.



Let’s calculate the force of tension in the string using the Newtons second law of motion:

$$\sum F_y = ma_y,$$

$$T - m_1 g = 0,$$

$$T = m_1 g = 10\,kg \cdot 9.8\,ms^{-2} = 98\,N.$$

Then, if we accelerate the system, the force of tension in the string will be the same as in part (a) – $T = 130.6\,N$. Therefore, we can conclude that the tension in the string increases when $m_1 > m_2$ and we accelerate the system of masses.