Answer on Question 58511, Physics, Mechanics, Relativity

Question:

The planet Jupiter has an elliptical orbit with $e = 0.05$ and a semi-major axis of $7.8 \cdot 10^{11}\ m$. Calculate the energy of the planet, perihelion and aphelion distances and the speed of planet at these points.

Solution:

a) We can find the energy of the planet from the formula:

$$E = \epsilon \frac{m_{Jupiter} M_{Sun}}{m_{Jupiter} + M_{Sun}},$$

here, ϵ is the specific orbital energy, $m_{Jupiter}$ is the mass of Jupiter, M_{Sun} is the mass of Sun. Let's write the formula, for the specific orbital energy:

$$\epsilon = -\frac{G\left(m_{Jupiter} + M_{Sun}\right)}{2a},$$

here, G is the gravitational constant, a is the semi-major axis.

Finally, we can substitute ϵ into the formula for the energy of the planet:

$$E = \epsilon \frac{m_{Jupiter} M_{Sun}}{m_{Jupiter} + M_{Sun}} = -\frac{G\left(m_{Jupiter} + M_{Sun}\right)}{2a} \cdot \frac{m_{Jupiter} M_{Sun}}{\left(m_{Jupiter} + M_{Sun}\right)} =$$
$$= -\frac{G m_{Jupiter} M_{Sun}}{2a}.$$

Let's substitute the numbers:

$$E = -\frac{6.672 \cdot 10^{-11}\ \frac{Nm^2}{kg^2} \cdot 1.8986 \cdot 10^{27}\ kg \cdot 1.988 \cdot 10^{30}\ kg}{2 \cdot 7.8 \cdot 10^{11}\ m} = -1.61 \cdot 10^{35}\ J.$$

b) We can find perihelion and aphelion distances from the first Kepler's law. It states, that all planets move in elliptical orbits, with the Sun at one focus. Then, applying the first Kepler's law we get:

$$r_p = a(1 - e),$$

$$r_a = a(1 + e),$$

here, r_p, r_a is the perihelion and aphelion distances, respectively; a is the semi-major axis of ellipse, e is the eccentricity of the ellipse.

Let's substitute the numbers:

$$r_p = a(1-e) = 7.8 \cdot 10^{11}\ m \cdot (1-0.05) = 7.4 \cdot 10^{11}\ m,$$

$$r_a = a(1+e) = 7.8 \cdot 10^{11}\ m \cdot (1+0.05) = 8.2 \cdot 10^{11}\ m.$$

c) We can find the speed of planet at these points from the vis-viva equation (also reffered to as orbital energy invariance law):

$$v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)},$$

here, G is the gravitational constant; M is the mass of Sun; r is the distance between Jupiter and Sun in perihelion and aphelion, respectively; a is the semi-major axis.

Then, for perihelion the speed of Jupiter will be:

$$v_p = \sqrt{6.672 \cdot 10^{-11}\ \frac{Nm^2}{kg^2} \cdot 2 \cdot 10^{30}\ kg \cdot \left(\frac{2}{7.4 \cdot 10^{11}\ m} - \frac{1}{7.8 \cdot 10^{11}\ m}\right)}$$
$$= 13768\ \frac{m}{s} = 13.768\ \frac{km}{s}.$$

For aphelion the speed of Jupiter will be:

$$v_a = \sqrt{6.672 \cdot 10^{-11}\ \frac{Nm^2}{kg^2} \cdot 2 \cdot 10^{30}\ kg \cdot \left(\frac{2}{8.2 \cdot 10^{11}\ m} - \frac{1}{7.8 \cdot 10^{11}\ m}\right)}$$
$$= 12425\ \frac{m}{s} = 12.425\ \frac{km}{s}.$$

Answer:

a) $E = -1.61 \cdot 10^{35}\ J$.

b) $r_p = 7.4 \cdot 10^{11}\ m$, $r_a = 8.2 \cdot 10^{11}\ m$.

c) $v_p = 13768\ \frac{m}{s}$, $v_a = 12425\ \frac{m}{s}$.