Answer on Question #58487, Physics / Mechanics | Relativity

A particle of mass m is projected with velocity u at angle x with horizontal. During the period when the particle descends from highest point to the position where its velocity vector makes an angle x/2 with horizontal. Work done by gravity force is - ?

Solution:

Initial condition:

A mass of particle m, and its initial velocity $\vec{u}$ and initial angle between $\vec{u}$ and horizontal equal x. In finish this angle is equal $\frac{x}{2}$.

$W = \vec{F} \cdot \vec{S}$, where $\vec{F} = m\vec{g}$ – gravitational force, where $\vec{g}$ – gravitational acceleration , and $\vec{S}$ – displacement vector.

$W-?$

$W = \vec{F} \cdot \vec{S} = F \cdot S \cdot \cos\theta = F \cdot \Delta h$.

Δh – displacement of the particle parallel vertical.

First velocity equal $\vec{u} = (v_{x0}; v_{y0}) = \vec{i}v_{x0} + \vec{j}v_{y0}$,

Where $v_{x0} = u\cos x$ and $v_{y0} = u\sin x$ projection vector on the x-axis and y-axis, respectively. $\vec{v}(t) = (v_x; v_y)$ velocity of the particle at time t.

We have only one force, which parallel y-axis, then $v_x = u\cos x = const$

$v_y = u\sin x - gt$.

We should be obtain work the particle descends from highest point to the position where its velocity vector makes an angle x/2 with horizontal

i. e. at $t = 0$ $v_x(0) = u\cos x$ and $v_y(0) = 0$ and

at moment t $v_x(t) = u\cos x$ and $v_y = -gt$. When the angle between velocity of particle and horizontal increased to $\frac{x}{2}$, we defined t_f - finish time.

After defined projection of displacement on y-axis Δh, from motion equation.

$\frac{v_y}{v_x} = \tan\theta$ – relation between the components of the velocity vector, When $t = t_f$, $\theta = -\frac{x}{2}$, because y-component of particle position decreising

$\tan\theta = \tan\left(-\frac{x}{2}\right) = \frac{v_y(t_f)}{v_x(t_f)} = \frac{-gt_f}{u\cos x}$

$-\tan\frac{x}{2} = -\frac{gt_f}{u\cos x}$

$\tan\frac{x}{2} = \frac{gt_f}{u\cos x}$

$t_f = \frac{u\cos x\tan\frac{x}{2}}{g}$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$$

$$t_f = \frac{u\cos x\tan\frac{x}{2}}{g} = \frac{u\cos x\sin x}{g(1+\cos x)}$$

$\varDelta h = v_y(0)\cdot t_f - \frac{gt_f^2}{2}$, $v_y(0) = 0$

$$\varDelta h = -\frac{gt_f^2}{2} = -\frac{gu^2\cos^2 x\sin^2 x}{g^2(1+\cos x)^2} = -\frac{u^2\cos^2 x\sin^2 x}{g(1+\cos x)^2}$$

"minus" sign means that displacement of the particle along the y-axis (height variation) directed downward, and the gravitational force acting on the particle, also aims to down.

Gravity $F = mg$ does work $W = mgh$ along any descending path

$$W = F_g\cdot\varDelta h = (-mg)\cdot\left(-\frac{u^2\cos^2 x\sin^2 x}{g(1+\cos x)^2}\right) = mg\cdot\frac{u^2\cos^2 x\sin^2 x}{g(1+\cos x)^2} =$$

$$= m\frac{u^2\cos^2 x\sin^2 x}{(1+\cos x)^2}$$

$\frac{\sin^2 x}{(1+\cos x)^2} = \tan^2\frac{x}{2}$:

$W = mu^2\cos^2 x\tan^2\frac{x}{2} = m\frac{u^2\cos^2 x\sin^2 x}{(1+\cos x)^2} = m\frac{u^2\sin^2 2x}{4(1+\cos x)^2}$

These three answer are equal.

Answer: $W = mu^2 \cos^2 x \tan^2 \frac{x}{2}$ or $W = m\frac{u^2 \cos^2 x \sin^2 x}{(1+\cos x)^2}$ or $W = m\frac{u^2 \sin^2 2x}{4(1+\cos x)^2}$