Answer on Question #58438, Physics / Mechanics | Relativity

a body of mass 100gm is suspended from the end of a light eleastic spring and set up into oscillatory motion.the equation of its displacement is x=2cos[2pie+pie/4]. find the phase constant and the period of oscillatory motion.what is the force acting on the body?

Find: φ – ? T – ? f – ?

Given:

m=100×10^{-3} kg

$x = 2\cos\left(2\pi t + \frac{\pi}{4}\right)$

Solution:

The equation of harmonic oscillations:

$x = x_{max}\cos(\omega t + \varphi_0)$ (1),

where $\varphi = \omega t + \varphi_0$ – phase constant

With the condition of the problem:

$x = 2\cos\left(2\pi t + \frac{\pi}{4}\right)$ (2)

Of (1) and (2) $\Rightarrow$ $\varphi = 2\pi t + \frac{\pi}{4}$

The cyclic frequency:

$\omega = \frac{2\pi}{T}$ (3),

where T – period of oscillatory motion

Of (3) $\Rightarrow$ $T = \frac{2\pi}{\omega}$ (4)

Of (1) and (2) $\Rightarrow$ $\omega = 2\pi$ (5)

(5) in (4): T=1 s

The equation of force:

$f = f_{max}\cos(\omega t + \varphi_0)$ (6),

where f_{max} – peak value of force

Newton's Second Law in scalar form:

$f = ma$ (7),

where a – acceleration of body

Of (7) $\Rightarrow$ $f_{max} = ma_{max}$ (8),

Acceleration of body:

$a = x_t''$ (9),

where x_t'' – the second derivative of the coordinates of time

Of (1) and (9) $\Rightarrow$

$a = (-\omega x_{max} \sin(\omega t + \varphi_0))' = -\omega^2 x_{max} \cos(\omega t + \varphi_0) = -a_{max} \cos(\omega t + \varphi_0)$ (10)

Of (10) $\Rightarrow a_{max} = \omega^2 x_{max}$ (11)

Of (2) $\Rightarrow x_{max} = 2$ m (12)

(5) and (12) in (11): $a_{max} = 8\pi^2 \ {}^{m}/_{s^2}$ (13)

(13) in (8): $f_{max} = 0{,}8\pi^2$ N (14)

(14) in (6): $f = 0{,}8\pi^2 \cos\left(2\pi t + \frac{\pi}{4}\right)$

Answer:

$\varphi = 2\pi t + \frac{\pi}{4}$

T=1 s

$f_{max} = 0{,}8\pi^2$ N

$f = 0{,}8\pi^2 \cos\left(2\pi t + \frac{\pi}{4}\right)$