Answer on Question 58365, Physics, Mechanics, Relativity

Question:

The speed of a spacecraft moving between Earth and Mars at an instant when Earth and Mars are $2.4 \cdot 10^{11}\ m$ apart is measured as $v = 0.8c$. The distance between the planets is measured in the fixed frame of reference in which Earth and Mars are at rest. What is the distance between Earth and Mars in the frame of the spacecraft? In the frame of the spacecraft, how much time elapses between the spacecraft crossing Earth and the spacecraft reaching Mars?

Solution:

a) We can find the distance between Earth and Mars in the frame of the spacecraft from the length contraction formula:

$$L = \frac{L_0}{\gamma} = L_0 \sqrt{1 - \frac{v^2}{c^2}},$$

here, L_0 is the proper length, v is the relative velocity between the observer and the moving object, c is the speed of light, $\gamma = 1/\sqrt{1 - \frac{v^2}{c^2}}$ is the Lorentz factor.

Therefore, we get:

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}} = 2.4 \cdot 10^{11}\ m \cdot \sqrt{1 - \frac{(0.8c)^2}{c^2}} = 2.4 \cdot 10^{11}\ m \cdot 0.6 = 1.44 \cdot 10^{11}\ m.$$

b) We can find, how much time elapses between the spacecraft crossing Earth and the spacecraft reaching Mars, from the formula:

$$\Delta t' = \frac{L}{v} = \frac{1.44 \cdot 10^{11}\ m}{0.8c} = \frac{1.44 \cdot 10^{11}\ m}{0.8 \cdot 3 \cdot 10^8\ \frac{m}{s}} = 600\ s.$$

Answer:

a) $L = 1.44 \cdot 10^{11}\ m$. b) $\Delta t' = 600\ s$.