Answer on Question 58260, Physics, Electromagnetism

Question:

A radio station broadcasts 50 kW (effective) from its transmitter at 810 kHz. Find the E_{rms} and B_{rms} for the electromagnetic radiation at a distance of 1 km from the transmitter. (Note: The give effective power assumes spherical symmetry; in practice the power is less because they send it out horizontally).

Solution:

a) Let's first write the formula for the Poynting vector:

$$\boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}, (1)$$

here, $\mu_0 = 4\pi \cdot 10^{-7}\ V \cdot s/A \cdot m$ is the vacuum permeability, $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic field.

We know, from the condition of the question, that the effective power assumes spherical symmetry, and this implies that the electromagnetic waves from the radio station radiates uniformly in all directions (though, we know that in practice the power is less because they send it out horizontally). Let's write the formula for the intensity of the electromagnetic radiation at the surface of the sphere:

$$|I| = \frac{P}{A_{surf}} = \frac{P}{4\pi r^2}, (2)$$

here, P is the effective power, $A_{surf} = 4\pi r^2$ is the surface area of the sphere, and r is the radius of the sphere.

It is obvious, that we can equate the equations (1) and (2) (because S and I both have the same units of dimension - W/m^2):

$$\frac{1}{\mu_0} E \times B = \frac{P}{4\pi r^2}.$$

We know, that $B = E/c$, therefore, substituting this expression into the previous formula we get:

$$\frac{1}{c\mu_0} E \times E = \frac{P}{4\pi r^2},$$

$$E^2 = \frac{Pc\mu_0}{4\pi r^2},$$

$$E_{rms} = \sqrt{\frac{Pc\mu_0}{4\pi r^2}} = \sqrt{\frac{50 \cdot 10^3\ W \cdot 3 \cdot 10^8\ \frac{m}{s} \cdot 4\pi \cdot 10^{-7}\frac{V \cdot s}{A \cdot m}}{4\pi \cdot (1000\ m)^2}} = 1.22\frac{V}{m}.$$

b) Then, the B_{rms} will be:

$$B_{rms} = \frac{E_{rms}}{c} = \frac{1.22\frac{V}{m}}{3 \cdot 10^8\ \frac{m}{s}} = 0.406 \cdot 10^{-8}T = 4.06nT.$$

Answer:

a) $E_{rms} = 1.22\frac{V}{m}$.

b) $B_{rms} = 4.06nT$.