Answer on Question #58208, Physics / Other

5. a) Classify the following PDEs by way of order and degree, linearity/nonlinearity, homogeneity/non-homogeneity:

i) d^2 u / dt^2 = d^2 u / dx^2 + x

ii) [1- (du/dt)^2] d^2 u / dx^2 - [1+ (du/dx)^2] d^2 u / dt^2 = 0

5. b) Show that the function f (x, y) = a(x^2 − y^2) + bxy where a and b are constants, is a solution of Laplace's equation.

5. c) Obtain all the first and second order partial derivatives of the function:

f(x,y) = y e^x - x^2 - y^-2 + tan y

Solution

a)i)

$$\frac{d^2u}{dt^2} = \frac{d^2u}{dx^2} + x$$

There are derivatives of second order, so PDE is second order
As all derivative are in first degree, so this PDE is first degree
This PDE is linear, because there is no coefficients which are functions of x,t,u
As there is x as separate part of equation, it is non-homogenous.

a)i)

$$\left(1 - (\frac{du}{dt})^2\right)\frac{d^2u}{dx^2} - \left(1 + (\frac{du}{dx})^2\right)\frac{d^2u}{dt^2} = 0$$

There are derivatives maximum of second order, so PDE is second order
As second derivative is in first degree, so this PDE is first degree
This PDE is nonlinear, because we have second degrees of first derivatives
As there is no independent variables (x,t) as separate part of equation, it is homogenous.

b) $f(x,y) = a(x^2 - y^2) + bxy$
Laplace's equation is:

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$$

$$\frac{d^2u}{dx^2} = \frac{d\frac{du}{dx}}{dx} = \frac{d(2ax + by)}{dx} = 2a$$

$$\frac{d^2u}{dy^2} = \frac{d\frac{du}{dy}}{dy} = \frac{d(-2ay + bx)}{dx} = -2a$$

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 2a - 2a = 0$$

So, function satisfies Laplace's equation

c) f(x,y) = y e^x - x^2 - y^-2 + tan y

$$f(x,y) = ye^x - x^2 - y^{-2} + tany$$

$$\frac{df}{dx} = ye^x - 2x$$

$$\frac{d^2f}{dx^2} = ye^x - 2$$

$$\frac{d^2f}{dxdy} = e^x$$

$$\frac{df}{dy} = e^x + 2y^{-3} + \frac{1}{cos^2y}$$

$$\frac{d^2f}{dy^2} = -6y^{-4} - + \frac{4siny}{cos^3y}$$