

Answer on Question #58101, Physics / Mechanics | Relativity

19 What is the gravitational field strength at a height h above the surface of the Earth? R is the radius of the earth

$gR^2(R+h)^2$

$gR(R+h)$

$g(R-h)R$

$g(R-h)^2R^2$

Solution:

On earth's surface [a distance of R away from the centre of Earth]

$$g = \frac{GM}{R^2}$$

So,

$$GM = gR^2$$

At h above ground [a distance of $(R+h)$ away from the centre of Earth]

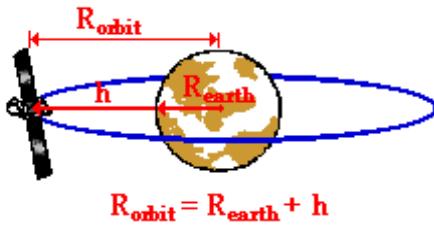
$$g = \frac{GM}{(R+h)^2} = \frac{gR^2}{(R+h)^2}$$

Answer: $\frac{gR^2}{(R+h)^2}$

20 A 2000 kg satellite orbits the earth at a height of 300 km. What is the speed of the satellite and its period? Take $G=6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$, Mass of the earth is $5.98 \times 10^{24} \text{ kg}$.

- 7.73 km/s and 5.4×10^3 s
- 855.4 km/s and 7.7×10^4 s
- 497.2 km/s and 5.5×10^5 s
- 322.3 km/s and 4.3×10^4 s

Solution:



This net centripetal force is the result of the gravitational force that attracts the satellite towards the central body and can be represented as

$$F_{\text{grav}} = \frac{(G * M_{\text{sat}} * M_{\text{earth}})}{R_{\text{orbit}}^2}$$

If the satellite moves in circular motion, then the net centripetal force acting upon this orbiting satellite is given by the relationship

$$F_{\text{net}} = \frac{(M_{\text{sat}} * v^2)}{R_{\text{orbit}}}$$

Since $F_{\text{grav}} = F_{\text{net}}$, the above expressions for centripetal force and gravitational force can be set equal to each other. Thus,

$$v^2 = \frac{(G * M_{\text{earth}})}{R_{\text{orbit}}}$$

The radius of earth is

$$R_{\text{earth}} = 6.37 \cdot 10^6 \text{ m}$$

Taking the square root of each side, leaves the following equation for the velocity of a satellite moving about a central body in circular motion

$$v = \sqrt{\frac{GM_{\text{earth}}}{R_{\text{earth}} + h}} = \sqrt{\frac{6.67 * 10^{-11} * 5.98 * 10^{24}}{6.37 * 10^6 + 300 * 10^3}} = 7733.1 \frac{\text{m}}{\text{s}} = 7.73 \frac{\text{km}}{\text{s}}$$

The final equation that is useful in describing the motion of satellites is Newton's form of Kepler's third law. Since the logic behind the development of the equation has been presented elsewhere, only the equation will be presented here. The period of a satellite (T) and the mean distance from the central body (R) are related by the following equation:

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM_{earth}}$$

where T is the period of the satellite, R is the average radius of orbit for the satellite.

Hence,

$$T = \sqrt{\frac{4\pi^2 R^3}{GM_{earth}}} = \sqrt{\frac{4 * \pi^2 * (6.37 * 10^6 + 300 * 10^3)^3}{6.67 * 10^{-11} * 5.98 * 10^{24}}} = 5419.45 \text{ s}$$

Answer: 7.73 km/s and 5.4×10³s.

