Answer on Question 58092, Physics, Astronomy, Astrophysics

Question:

The change in the value of acceleration of Earth towards Sun, when the Moon comes from the position of solar eclipse to the position on the other side of Earth in line with the Sun is? (mass of Moon is $7.36 \cdot 10^{22}\ kg$, orbital radius of Moon is $3.82 \cdot 10^{8}\ m$)

Solution:

Let's draw, for clarity, the free-body diagrams for Earth in the two situations (solar eclipse and lunar eclipse):

Here, F_S is the magnitude of the gravitational attraction of the Sun on Earth, F_M is the magnitude of the gravitational attraction of the Moon on the Earth. It is obvious, that the magnitudes of the net forces acting on the Earth are $F_S + F_M$ in case of a solar eclipse and $F_S - F_M$ in case of a lunar eclipse.

Let's write the forces F_S and F_M using the Newton's Law of Universal Gravitation:

$$F_S = G\frac{M_S M_E}{D_S^2}, \quad F_M = G\frac{M_M M_E}{D_M^2},$$

here, G is the universal gravitational constant, M_S is the mass of the Sun, M_E is the mass of the Earth, M_M is the mass of the Moon, D_S is the distance from the Earth to the Sun, D_M is the distance from the Earth to the Moon.

It is obviously, that the net forces acting on the Earth in cases of the solar eclipse and the lunar eclipse are balanced by the Earth's acceleration $M_E a$:

$$M_E a_S = \sum F = F_S + F_M, \quad M_E a_L = \sum F = F_S - F_M,$$

$$M_E a_S = G\frac{M_S M_E}{D_S^2} + G\frac{M_M M_E}{D_M^2}, \qquad M_E a_L = G\frac{M_S M_E}{D_S^2} - G\frac{M_M M_E}{D_M^2},$$

$$a_S = G\left(\frac{M_S}{D_S^2} + \frac{M_M}{D_M^2}\right), \quad a_L = G\left(\frac{M_S}{D_S^2} - \frac{M_M}{D_M^2}\right),$$

here, a_S is the acceleration of Earth in case of the solar eclipse, a_L is the acceleration of Earth in case of the lunar eclipse.

Then, we can find the change in the value of acceleration of Earth towards Sun, when the Moon comes from the position of solar eclipse to the position on the other side of Earth in line with the Sun:

$$a_S = G\left(\frac{M_S}{D_S^2} + \frac{M_M}{D_M^2}\right) = 6.67 \cdot 10^{-11}\ \frac{N \cdot m^2}{kg^2} \cdot \left(\frac{1.99 \cdot 10^{30}\ kg}{(1.50 \cdot 10^{11}\ m)^2} + \frac{7.36 \cdot 10^{22}\ kg}{(3.82 \cdot 10^{8}\ m)^2}\right) = 0.00593288\ \frac{m}{s^2}.$$

$$a_L = G\left(\frac{M_S}{D_S^2} - \frac{M_M}{D_M^2}\right) = 6.67 \cdot 10^{-11}\ \frac{N \cdot m^2}{kg^2} \cdot \left(\frac{1.99 \cdot 10^{30}\ kg}{(1.50 \cdot 10^{11}\ m)^2} - \frac{7.36 \cdot 10^{22}\ kg}{(3.82 \cdot 10^{8}\ m)^2}\right) = 0.0058656\ \frac{m}{s^2}.$$

$$a_S - a_L = 0.00593288\ \frac{m}{s^2} - 0.0058656\ \frac{m}{s^2} = 0.0000673\ \frac{m}{s^2} = 6.73 \cdot 10^{-5}\ \frac{m}{s^2}.$$

Answer:

$$a_S - a_L = 6.73 \cdot 10^{-5}\ \frac{m}{s^2}.$$