

**Answer on** Question #57786, Physics / Mechanics | Relativity

A sailor pushes a 100.0 kg crate up a ramp that is 3.00 m high and 5.00 m long onto the deck of a ship. He exerts a 650.0 N force parallel to the ramp. What is the mechanical advantage of the ramp? What is the efficiency of the ramp? Your response should include all of your work and a free-body diagram.

**Find:**  $\Delta F - ? \eta - ?$

**Given:**

$$m=100 \text{ kg}$$

$$h=3 \text{ m}$$

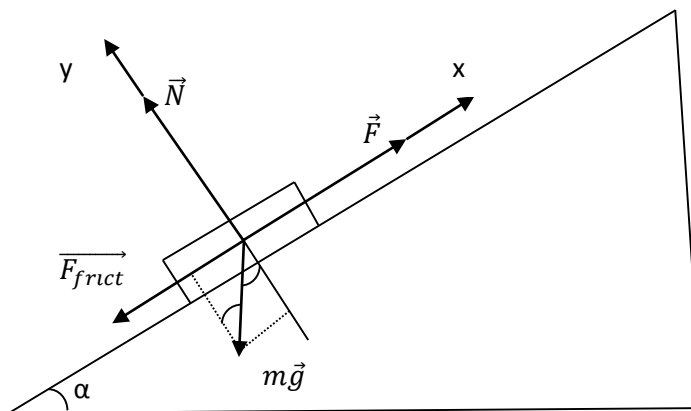
$$l=5 \text{ m}$$

$$F=650 \text{ N}$$

$$g=9,8 \text{ N/kg}$$

**Solution:**

Consider the forces, which acting on the crate.



Newton's Second Law:

$$\sum_{i=1}^n \vec{F}_i = m\vec{a} \quad (1)$$

We believe that the body moves in straight lines and uniformly.

$$\text{Because } \vec{a} = \vec{0} \quad (2)$$

Write the vector sum of all forces:

$$\sum_{i=1}^n \vec{F}_i = \vec{F} + \vec{F}_{frict} + \vec{N} + m\vec{g} \quad (3),$$

where  $\vec{F}$  – traction force,

$\vec{F}_{frict}$  – friction force,

$\vec{N}$  – reaction force,

$m\vec{g}$  – gravity

(2) and (3) in (1):

$$\vec{F} + \vec{F}_{\text{frict}} + \vec{N} + m\vec{g} = \vec{0} \quad (4)$$

Find the projection of forces.

$$\text{OX: } F - F_{\text{frict}} - mg \sin \alpha = 0 \quad (5)$$

$$\text{OY: } N - mg \cos \alpha = 0 \quad (6)$$

Friction force (with a particular approach):

$$F_{\text{frict}} = \mu N \quad (7),$$

where  $\mu$  – coefficient of friction ( $\mu < 1$ )

$$\text{Of (6)} \Rightarrow N = mg \cos \alpha \quad (8)$$

$$(8) \text{ in (7): } F_{\text{frict}} = \mu mg \cos \alpha \quad (9)$$

$$\text{Of (5)} \Rightarrow F = F_{\text{frict}} + mg \sin \alpha \quad (10)$$

$$(9) \text{ in (10): } F = mg(\mu \cos \alpha + \sin \alpha) \quad (11)$$

$$\text{Expression } (\mu \cos \alpha + \sin \alpha) < 1 \quad (12)$$

$$\text{Of (11) and (12)} \Rightarrow F = mg(\mu \cos \alpha + \sin \alpha) < mg \quad (13)$$

Of (13)  $\Rightarrow$  mechanical advantage of the ramp:

$$\Delta F = mg - F \quad (14)$$

$$\text{Of } \Rightarrow \Delta F = 330 \text{ N}$$

Efficiency of the ramp:

$$\eta = \frac{A_{\text{helpful}}}{A_{\text{spent}}} \times 100\% \quad (15),$$

where  $A_{\text{helpful}}$  – helpful work,

$A_{\text{spent}}$  – spent work

$$\text{Helpful work: } A_{\text{helpful}} = mgh \quad (16),$$

$$\text{Spent work: } A_{\text{spent}} = Fl \quad (17)$$

(16) and (17) in (15):

$$\eta = \frac{mgh}{Fl} \times 100\% \quad (18)$$

$$\text{Of (18)} \Rightarrow \eta = 90\%$$

**Answer:**

$$\Delta F = 330 \text{ N}$$

$$\eta = 90\%$$