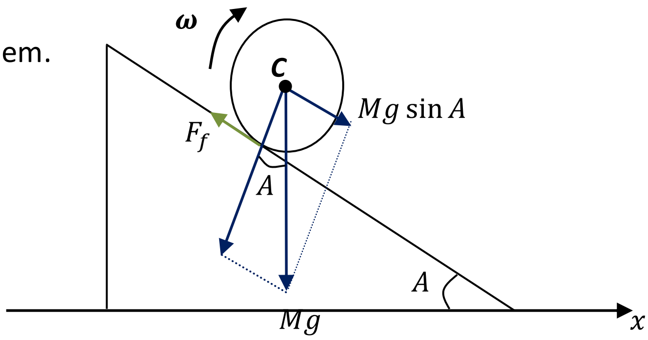
Answer on Question#57637 –Mechanics – Relativity

There is a cylinder on inclined plane which has a angle of 'A' with x-axis. Prove that the minimum friction force required to roll the cylinder without slipping is "2/3MgsinA". 'M' is the mass of the cylinder 'g' is the acceleration due to gravity.

Solution.

Construct a pattern to the condition of the problem.
C – the center of mass of the cylinder.
Mg – force of gravity.
F_f – friction force.



Projection of the force of gravity on an inclined plane is equal to $Mg\sin A$.
The general relationship among torque, moment of inertia, and angular acceleration is

$$I\varepsilon = netT$$

I – moment of inertia, ε – angular acceleration, $netT$ – is the total torque from all forces relative to a chosen axis. Consider the equation with respect to the axis passing through the center of mass. In our case only torque of friction force is not zero. Torque of friction force equal F_fR, where R – radius of the cylinder. Moment of inertia of the cylinder $I = \frac{MR^2}{2}$.
Consider Newton's second law

$$Ma = Mg\sin A - F_f \text{ or } M\frac{dv}{dt} = Mg\sin A - F_f.$$

The general relationship among acceleration and angular acceleration is

$$a = \frac{dv}{dt} \to a = \frac{d(\omega R)}{dt} \to a = R\frac{d\omega}{dt} \to a = \varepsilon R.$$

Hence

$I\frac{a}{R} = F_fR \to \; a = \frac{F_fR^2}{I} \to a = \frac{2F_fR^2}{MR^2} \to \; a = \frac{2F_f}{M}.$

Substituting these expression into the equation $Ma = Mg\sin A - F_f$ get

$M\frac{2F_f}{M} = Mg\sin A - F_f \to 2F_f = Mg\sin A - F_f \to 3F_f = Mg\sin A \to F_f = \frac{1}{3}Mg\sin A.$

Answer: $F_f = \frac{1}{3}Mg\sin A.$