Answer on Question#57550 - Physics - Mechanics - Relativity

A thin rod length $2l$ and mass M is located above the flat surface slippery. A small ball mass m and velocity v_0 pounding the rod tip. Assume that the collision is elastic and the speed of the ball after a collision parallel with the initial speed.

A) Calculate the ratio of M/m so that the ball is at rest after the collision

B) Calculate the minimum time that required of the rod to perform one round

C) Calculate the speed of the ball if there is a shaft at the lower end of the rod

Solution:

A) According to the law of conservation of momentum:

$$mv_0 = Mv,$$

Where v—is the final velocity of the rod.

Thus,

$$v = \frac{m}{M}v_0$$

The moment of inertia of the rod about its center is

$$I = \frac{1}{3}Ml^2$$

According to the law of conservation of angular momentum (about center of the rod):

$$mv_0 l = I\omega,$$

Where ω – is the angular speed of the rod after collision.

Thus,

$$\omega = 3\frac{m}{M}\frac{v_0}{l}$$

According to the law of conservation of energy we obtain

$$\frac{mv_0^2}{2} = \frac{I\omega^2}{2} + \frac{Mv^2}{2}$$

$$\frac{mv_0^2}{2} = \frac{3}{2}\frac{m^2}{M}v_0^2 + \frac{1}{2}\frac{m^2}{M}v_0^2$$

Therefore

$$\frac{M}{m} = 4$$

B) The angular speed of the rod after collision

$$\omega = \frac{3}{4}\frac{v_0}{l}$$

Period

$$T = \frac{2\pi}{\omega} = \frac{8\pi l}{3v_0}$$

C) The moment of inertia of the rod about one of the tips is

$$I_t = \frac{4}{3}Ml^2$$

Thus, according to the law of conservation of angular momentum we obtain (the ball must be at rest after collision)

$$I_t\omega_t = mv_0 l$$

$$\frac{4}{3}ml^2\omega_t = mv_0 l$$

$$\omega_t = \frac{3}{4}\frac{v_0}{l}$$

According to the law of conservation of energy we obtain

$$\frac{I_t\omega_t^2}{2} = \frac{mv_0^2}{2}$$

$$\frac{3}{4}Mv_0^2 = mv_0^2$$

Therefore

$$m = \frac{3}{4}M$$

Answer:

A) 4

B) $\frac{8\pi l}{3v_0}$

C) $\frac{3}{4}M$