Answer on Question 57424, Physics, Other

Question:

120 atoms: 40 have decayed, what is the half-life? What is it in years?

Solution:

Given: beginning amount of atoms – 120 atoms; 40 have decayed means the ending amount of atoms is $120 - 40 = 80$ atoms.

Let's first calculate number of half-lives, n from the formula:

$$Ending\ Amount = \frac{Beginning\ Amount}{2^n},$$

$$\left(\frac{1}{2}\right)^n = \frac{Ending\ Amount}{Beginning\ Amount} = \frac{80}{120}.$$

Let's take the log of both sides of equation:

$$log\left(\frac{1}{2}\right)^n = \log\left(\frac{80}{120}\right),$$

$$n \cdot \log(0.5) = \log\left(\frac{80}{120}\right),$$

$$n = \log\left(\frac{80}{120}\right)/\log(0.5) = 0.585.$$

In order to find the half-life we must know the elapsed time. We can find it from the formula:

$$Beginning\ Amount \cdot \left(\frac{1}{2}\right)^{\left(\frac{Elaps.time}{n}\right)} = Ending\ Amount,$$

$$120 \cdot \left(\frac{1}{2}\right)^{\left(\frac{Elaps.time}{0.585}\right)} = 80,$$

$$\left(\frac{1}{2}\right)^{\left(\frac{Elaps.time}{0.585}\right)} = \frac{80}{120}.$$

Again take the log of both sides of equation:

$$log\left(\frac{1}{2}\right)^{\left(\frac{Elaps.time}{0.585}\right)} = \log\left(\frac{80}{120}\right),$$

$$\log(0.5)\cdot\left(\frac{Elaps.time}{0.585}\right) = \log\left(\frac{80}{120}\right),$$

$$Elaps.time = 0.585\cdot\frac{\log\left(\frac{80}{120}\right)}{\log(0.5)} = 0.342year.$$

Then we can find the half-life from the formula:

$$T_{1/2} = \frac{Elaps.time\cdot\log 2}{log\left(\frac{Beginning\ Amount}{Ending\ Amount}\right)} = \frac{0.342year\cdot\log 2}{log\left(\frac{80}{120}\right)} = 0.584year.$$

Answer:

$T_{1/2} = 0.584year.$