Answer on Question #57394-Physics – Mechanics | Relativity

A football placekicker consistently kicks a football with a resultant velocity of 23 m/s at an angle of 43º relative to the horizontal. What is the maximum distance from the goal post the kicker can make a field goal? Assume the bottom bar of the goal post is 10 feet (3.05 m) off of the ground.

Solution

There are two unknowns, the time it takes the football to reach the bottom bar of the goal posts and the distance from the goal post the kicker can make a field goal. We'll write the horizontal and vertical trajectory equations and eliminate the time dependence and determine the distance from the goal post the kicker can make a field goal.

$$x = v\ \cos\theta\, t \to t = \frac{x}{v\ \cos\theta}$$

$$y = v\ \sin\theta\, t - \frac{gt^2}{2} = v\ \sin\theta \frac{x}{v\ \cos\theta} - \frac{g}{2}\left(\frac{x}{v\ \cos\theta}\right)^2 = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta},$$

$$\frac{gx^2}{2v^2\cos^2\theta} - x\tan\theta + y = 0.$$

$$D = \tan^2\theta - 4y\frac{g}{2v^2\cos^2\theta}.$$

The maximum distance from the goal post the kicker can make a field goal is

$$x_{max} = \frac{\tan\theta + \sqrt{\tan^2\theta - 2y\frac{g}{v^2\cos^2\theta}}}{\frac{g}{v^2\cos^2\theta}} = \frac{\tan 43 + \sqrt{\tan^2 43 - 2\cdot 3.05\cdot\frac{9.8}{23^2\cos^2 43}}}{\frac{9.8}{23^2\cos^2 43}} = 50\ m.$$

Answer: $50\ m$.