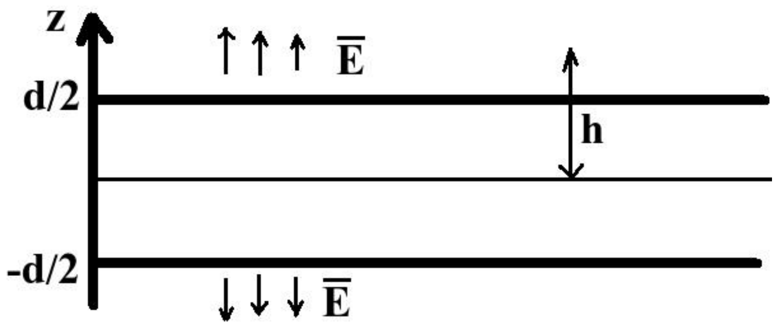
Answer to the question #57331, Physics / Electromagnetism

An electric charge is uniformly distributed with volume density ρ = 10^{-8} C/m3 between two infinite parallel plates which are d= 2 cm apart. Calculate the electric flux density D(x) due to this charge distribution and plot schematically its dependence on x. Now imagine that a point charge q = 1.5 × 10^{-8} C is placed in the median plane (the plane parallel to the two plates, midway between them) of the charge distribution and determine what is the work that the electrostatic force has to do in order to move q to a point P, located outside the region where the charge is distributed, at a distance h = 3 cm from the nearest plate.

Answer:

d=2cm, h=3 cm, q = 1.5 × 10^{-8} C, ρ = 10^{-8} C/m3

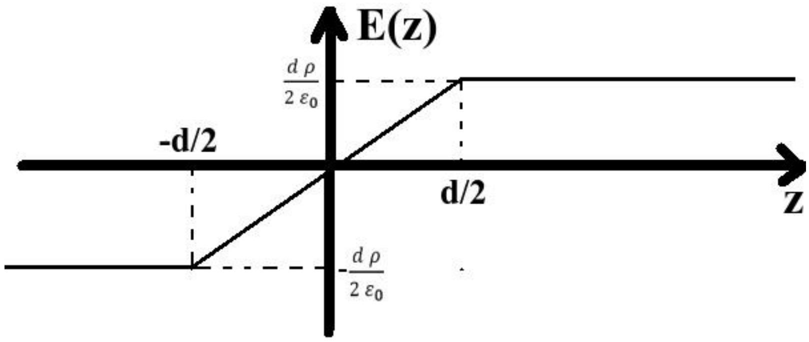


It follows from the symmetry of the system that electric field is perpendicular to plates. E also depends on z only. Suppose, we have two planes which are 2z apart. They characterize the electric field E(z) and E(-z)=E(z). According to the Gauss theorem $\int_s \vec{E}(z)ds = 2\,S\,E(z) = \frac{Q(z)}{\varepsilon_0}$;

$Q(z)$- electric charge between planes which are 2z apart.

$Q(z) = s\,\mathrm{d}\,\rho$ → $E(z) = \frac{d\,\rho}{2\,\varepsilon_0}$ for z ≥ d/2;

$Q(z) = 2\,s\,\mathrm{z}\,\rho$ → $E(z) = \frac{z\,\rho}{\varepsilon_0}$ for z < d/2;



$$A = \int \vec{F}\, d\vec{r} = \int \vec{E} q\, d\vec{r} = \int_0^h E(z)\, q\, dz = q \left(\int_0^{d/2} E(z)\, dz + \int_{d/2}^{h} E(z)\, dz \right)$$

$$= \frac{q\, \rho}{\varepsilon_0} \left(\int_0^{d/2} z\, dz + \int_{d/2}^{h} \frac{d}{2}\, dz \right) = \frac{q\, \rho}{\varepsilon_0} \left(\frac{d^2}{8} + \frac{d}{2}\left(h - \frac{d}{2}\right) \right) = \frac{q\, d\, \rho}{2\, \varepsilon_0} \left(h - \frac{d}{4}\right)$$

$$= \frac{1.5 \times 10^{-8} \times 0.02 \times 10^{-8}}{2 \times 8.8541 \times 10^{-12}} (0.03 - 0.005) = 4.235 \times 10^{-9} J$$