Answer on Question 57235, Physics, Other

Question:

Find Laplace transform of $f(t) = e^{2t}$.

Solution:

Let's find the Laplace transform of $f(t) = e^{2t}$. By the definition of the Laplace transform we have:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t)dt = \lim_{n\to\infty} \int_0^{n} e^{-st} f(t)dt.$$

Substituting our function into the definition of the Laplace transform we obtain:

$$F(s) = \mathcal{L}\{f(t)\} = \lim_{n\to\infty} \int_0^{n} e^{-st}\, e^{2t} dt = \lim_{n\to\infty} \int_0^{n} e^{-(s-2)t}\, dt = \lim_{n\to\infty} -\frac{1}{s-2} e^{-(s-2)t} \Big|_0^n$$

$$= \lim_{n\to\infty} -\frac{1}{s-2}\left(e^{-(s-2)n} - 1\right) = \frac{1}{s-2}.$$

Using the Table of Laplace transforms we can check, whether we find the Laplace transform of function $f(t) = e^{2t}$ correctly:

$$F(s) = \mathcal{L}\{e^{\alpha t}\} = \frac{1}{s-\alpha}.$$

Then,

$$F(s) = \mathcal{L}\{e^{2t}\} = \frac{1}{s-2}.$$

So, we find the Laplace transform of $f(t) = e^{2t}$ correctly.

Answer:

$$F(s) = \frac{1}{s-2}.$$