Answer on Question #57234, Physics / Mechanics | Relativity | for completion

A pillar of circular cross section is 5 m long. A 200 kN load acts at the top of the pillar. Determine maximum stress and total shortening of the pillar due to the top loadand its own weight. The unit weight of the material is 79 kN/m^3 and modulus of elasticity, E = 2×10^5 N/mm^2.

Find: σ – ? ε – ? Δl – ? P – ?

Given:

l_0=5 m

F=200×10^3 N

$\rho=8$×10^3 kg/m^3

E=2×10^5 N/mm^2=2×10^{11} N/m^2

$g=9{,}8$ N/kg

Solution:

The deformation is elastics.

The relationship between the density of matter and unit weight of the material:

$\rho = \frac{m}{V} = \frac{P}{Vg}$ (1),

where ρ – density of matter, which made the pillar,

m – mass of column,

V – volume of column,

P – weight of column,

g – acceleration of gravity,

$\frac{P}{V}$ – unit weight of the column's material.

Calculation of mechanical stress, which occurs due to the own weight of the column

Mechanical stress: $\sigma_0 = \frac{F_{\text{elast}}}{S}$ (2),

where F_{elast} – strength elasticity (numerically equal to the weight of column P),

S – sectional area of the column.

Mass of column: $m = \rho V$ (3),

where ρ – density of matter, from which is made the pillar,

V – volume of column.

The column has a cylindrical shape.

Volume of column: $V = Sl_0$ (4),

where l_0 – length of column.

(4) y (3): $m = \rho S l_0$ (5)

З (5) ⇒ $S = \frac{m}{\rho l_0}$ (6)

Weight of column: $P = mg$ (7)

З (7) ⇒ $m = \frac{P}{g}$ (8)

(8) y (6): $S = \frac{P}{\rho l_0 g}$ (9)

(9) y (2): $\sigma_0 = \frac{P\rho l_0 g}{P} = \rho l_0 g$ (10)

З (10) ⇒ $\sigma_0 = 3{,}92 \times 10^5\ Pa$ (11).

Calculation of mechanical stress, which occurs due to the additional load

$\sigma_1 = \frac{F_{1\,\mathrm{elast}}}{S}$ (12),

where F_{1elast} – strength elasticity (numerically equal to the force F)

S – sectional area of the column.

З (1) ⇒ $\frac{P}{V} = \rho g$ (13)

(4) y (13): $\frac{P}{Sl_0} = \rho g$ (14)

З (14) ⇒ $S = \frac{P}{l_0 \rho g}$ (15)

З (15) ⇒ $S = 0{,}2 m^2$ (16)

(16) y (12): $\sigma_1 = 1 \times 10^6 Pa$ (17).

General mechanical stress: $\sigma = \sigma_0 + \sigma_1$ (18)

(11) i (17) y (18): $\sigma = 1{,}392 \times 10^6 Pa$ (19).

Hooke's Law: $\sigma = E\varepsilon$ (20),

where σ – general mechanical stress,

E – Young's modulus,

ε – relative compression.

З (20) ⇒ $\varepsilon = \frac{\sigma}{E}$ (21)

3 (21) $\Rightarrow$ $\varepsilon = 6{,}96 \times 10^{-4}\%$ (22).

Relative compression: $\varepsilon = \frac{\Delta l}{l_0}$ (23),

where Δl – absolute compression,

l_0 – initial length of column.

3 (23) $\Rightarrow$ $\Delta l = \varepsilon l_0$ (24)

3 (24) $\Rightarrow$ $\Delta l = 3{,}48 \times 10^{-6} m$ (24).

Weight of column: $P = mg$ (25)

(5) y (25): $P = \rho S l_0 g$ (26)

(16) y (26): $P = 78{,}4 \times 10^3 kg$.

Answer:

$\sigma = 1{,}392 \times 10^6 Pa$,

$\varepsilon = 6{,}96 \times 10^{-4}\%$,

$\Delta l = 3{,}48 \times 10^{-6} m$,

$P = 78{,}4 \times 10^3 kg$.