boat 'a' is moving with speed of 10 km/h in west direction. in 100 km south of a is boat 'b' moving towards north (speed of boat 'b' is also 10 km/h). in what time will the two boats be at the shortest distance from each other

Solution:
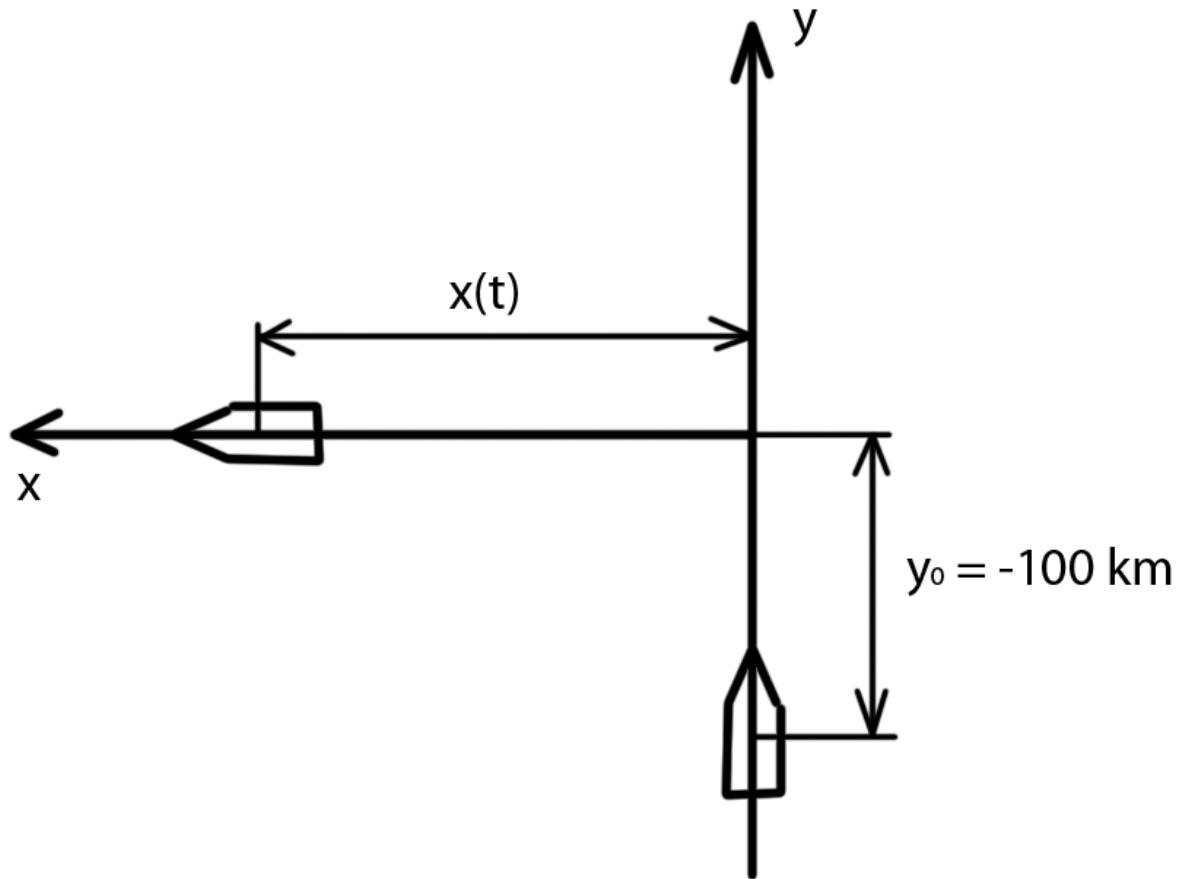
Distance between two boats:

$$d = \sqrt{x^2(t) + y^2(t)} \tag{1}$$

Displacement of boat 1:

$$x(t) = 10t \tag{2}$$

Displacement of boat 2:

$$y(t) = -100 + 10t \tag{3}$$

Combining (1), (2) and (3):

$$d = \sqrt{100t^2 + (10t - 100)^2} \ ;$$

$$d = 10\sqrt{2t^2 - 10t + 100} \tag{4}$$

To find the smallest distance between the boats, one should minimize function (4). Considering the interval 0 <= t <= 10, because after 10 hours boat 2 will reach x-axis and distance will be 100 km (displacement of boat 1 during 10 hours), and will grow further, as seen from the graph.

$$d' = \frac{20 \times (2t - 5)}{\sqrt{2t^2 - 10t + 100}} \ ;$$

$d' = 0$;

$$\frac{20 \times (2t - 5)}{\sqrt{2t^2 - 10t + 100}} = 0 \;;$$

t = 2.5; d(2.5) = 93.54 km

Answer: in 2.5 hours after initial moment.

$d' = 0$;

$$\frac{20 \times (2t - 5)}{\sqrt{2t^2 - 10t + 100}} = 0 \;;$$