Answer on Question 56960, Physics, Mechanics, Relativity

Question:

A turntable with a moment of inertia of $0.011 kg \cdot m^2$ rotates freely at $3.1\, rad/s$. A circular disk of mass $400g$ and diameter of $22cm$, and initially not rotating, slips down a spindle and lands on the turntable.

a) Find the new angular speed (rad/s).

b) What is the change in kinetic energy?

Solution:

a) We can find the new angular speed of the turntable from the law of conservation of angular momentum:

$$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega,$$

here, $I_1\omega_1$ is the initial angular momentum of the turntable, $I_2\omega_2$ is the initial angular momentum of the circular disk (since the circular disk initially not rotating, $I_2\omega_2 = 0\frac{kg\cdot m^2}{s}$), I_1, I_2 are the moments of inertia of the turntable and circular disk, respectively, ω_1, ω_2 are the angular speeds of the turntable and circular disk, respectively and ω is the new angular speed.

By the definition of the moment of inertia of the circular disk we have:

$$I_2 = \frac{1}{2}m_2r^2 = \frac{1}{2}\cdot 0.4kg \cdot (0.11m)^2 = 2.42\cdot 10^{-3} kg\cdot m^2.$$

Then, we can find the new angular speed of the turntable:

$$\omega = \frac{I_1\omega_1 + I_2\omega_2}{(I_1 + I_2)} = \frac{0.011kg\cdot m^2 \cdot 3.1\frac{rad}{s} + 2.42\cdot 10^{-3}kg\cdot m^2\cdot 0\frac{rad}{s}}{0.011kg\cdot m^2 + 2.42\cdot 10^{-3}kg\cdot m^2} =$$

$$= \frac{0.0341kg\cdot m^2\cdot\frac{rad}{s}}{0.01342kg\cdot m^2} = 2.54\frac{rad}{s}.$$

b) Let's write the initial kinetic energy of the turntable and the circular disk:

$$KE_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}\cdot 0.011kg\cdot m^2\cdot\left(3.1\frac{rad}{s}\right)^2 = 0.053J.$$

$$KE_2 = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} \cdot 2.42 \cdot 10^{-3} kg \cdot m^2 \cdot \left(0 \frac{rad}{s}\right)^2 = 0J.$$

The final kinetic energy after the circular disk lands on the turntable will be:

$$KE_{final} = \frac{1}{2} I_1 \omega^2 + \frac{1}{2} I_2 \omega^2 = \frac{1}{2} (I_1 + I_2) \omega^2 = \frac{1}{2} (I_1 + I_2) \cdot \frac{(I_1 \omega_1 + I_2 \omega_2)^2}{(I_1 + I_2)^2}$$

$$= \frac{1}{2} \frac{(I_1 \omega_1 + I_2 \omega_2)^2}{(I_1 + I_2)} = \frac{(0.011 kg \cdot m^2 \cdot 3.1 \frac{rad}{s})^2}{2 \cdot (0.011 kg \cdot m^2 + 2.42 \cdot 10^{-3} kg \cdot m^2)} =$$

$$= 0.043J.$$

Then, the change in kinetic energy will be:

$$\Delta KE = KE_{final} - (KE_1 + KE_2) = 0.043J - 0.053J = -0.01J.$$

Sign minus means that we have the loss in kinetic energy.

Answer:

a) $\omega = 2.54 \frac{rad}{s}$.

b) $\Delta KE = 0.01J$.