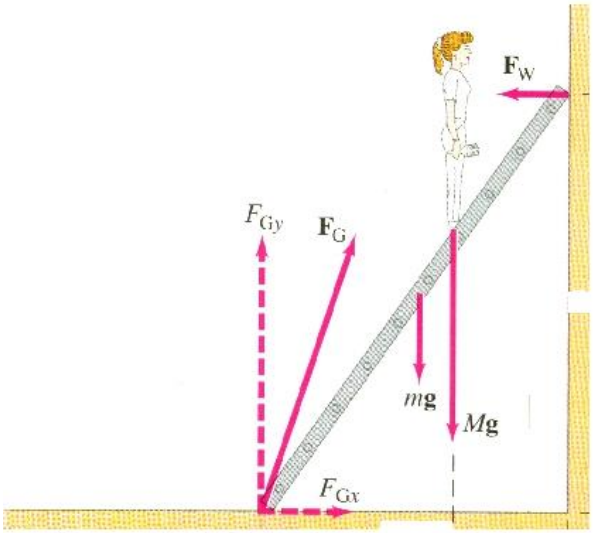
Answer on Question #56953, Physics / Mechanics | Relativity

A ladder of mass M and length 4 m rests against a frictionless wall at an angle of 50 degrees to the horizontal. The coefficient of static friction between the ladder and the floor is 0.65. What is the maximum distance along the ladder a person of mass 15M can climb before the ladder starts to slip?

Solution:



We know at the point of slipping that F_{Gx} (See diagram above) is just equal to the friction force, where the normal force is F_{Gy} (The upward force exerted by the ground on the ladder - see diagram above.)

So, applying the formula for friction:

$F_{fr} = m_s F_n$ (at the point of slipping)

where m_s is the coefficient of static friction between the ladder and the floor

$$F_{Gx} = m_s F_{Gy}$$

Vertical Force:

Ladder weight: $-Mg$ (down)

Person weight: $-15Mg$ (down)

Ground pushing up: $+F_{Gy}$ (up)

Equilibrium:

$$F_{Gy} - Mg - 15Mg = 0$$
$$F_{Gy} - 16Mg = 0$$

Thus,

$$F_{Gy} = 16Mg$$

Horizontal Force:

Force exerted by the wall: $-F_W$ (left)

Force exerted by the ground horizontally: $+F_{Gx}$ (right)

Equilibrium:

$$F_{Gx} - F_W = 0$$
$$F_{Gx} = F_W = 0.65 \cdot 16Mg = 10.4Mg$$

And finally torque:

We then have the following torques about the bottom of the ladder:

The ground: torque = 0 (r = 0)

The weight of the ladder: Mg N at an angle of 50° at a distance of 4/2=2 m from the bottom of the ladder. $Torque_1 = (2.0\ m)(Mg\ N)\cos(50°)$ (CW)

The weight of the person on the ladder: 15Mg N at an angle of 50° at a distance of x m from the bottom of the ladder. $Torque_2 = x \cdot 15Mg \cdot \cos(50°)$ (CW)

The wall pushing to the left: F_W acting at a distance of 4.0 m from the bottom, at an angle of 50° with the ladder.

$Torque_3 = -(4.0\ m)F_W sin(50°)$ (ACW)

Equilibrium:

$$Torque_1 + Torque_2 + Torque_3 = 0$$

$$2Mg\cos(50°) + 15xMg\cos(50°) - 4 \cdot 10.4Mgsin(50°) = 0$$

$$2\cos(50°) + 15x\cos(50°) - 41.6\sin(50°) = 0$$

$$x = \frac{41.6\sin(50°) - 2\cos(50°)}{15\cos(50°)} = 1.311\ \text{m}$$

Answer: 1.311 m