Answer on Question#56938 - Physics – Mechanics | Relativity

Two balls of equal masses are projected upward simultaneously, one from the ground with speed $v_1 = 50\ \mathrm{m/s}$ and other from a $h_0 = 40\ \mathrm{m}$ high tower with initial speed $v_2 = 30\ \mathrm{m/s}$. Find the maximum height attained by their centre of mass.

<u>Solution:</u>

The dependence of the height on time of the first ball is given by

$$h_1(t) = h_0 + v_1 t - \frac{gt^2}{2}$$

The dependence of the height on time of the second ball is given by

$$h_2(t) = v_2 t - \frac{gt^2}{2}$$

Since masses of balls are equal, the dependence of the height on time of the centre of mass is given by

$$h_c(t) = \frac{h_1(t) + h_2(t)}{2} = \frac{h_0}{2} + \frac{v_1 + v_2}{2} t - \frac{gt^2}{2}$$

It reaches maximum height when $h_c'(t) = 0$:

$$\frac{d}{dt}\left(\frac{h_0}{2} + \frac{v_1 + v_2}{2} t - \frac{gt^2}{2}\right) = 0$$

$$\frac{v_1 + v_2}{2} - gt = 0$$

$$t = \frac{v_1 + v_2}{2g}$$

Thus the maximum height attained by centre of mass is

$$h_{\max} = h_c\left(\frac{v_1 + v_2}{2g}\right) = \frac{h_0}{2} + \frac{1}{g}\left(\frac{v_1 + v_2}{2}\right)^2 - \frac{g\left(\frac{v_1 + v_2}{2g}\right)^2}{2} = \frac{h_0}{2} + \frac{(v_1 + v_2)^2}{8g} =$$

$$= \frac{40\ \mathrm{m}}{2} + \frac{\left(50\,\frac{\mathrm{m}}{\mathrm{s}} + 30\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{8 \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 101.6\ \mathrm{m}$$

<u>Answer:</u> $101.6\ \mathrm{m}$.