Answer on Question#56931 - Physics - Mechanics - Relativity

A body of mass $m = 2$ kg is moving under the influence of a central force whose potential energy is given by $U(r) = 2r^3$ Joule. If the body is moving in a circular orbit of $R = 5\text{m}$, then find its energy.

Solution:

The force of potential field

$$\boldsymbol{F}(r) = -\nabla U(r) = -6r^2\frac{\boldsymbol{r}}{r}$$

Since the body is moving in a circular orbit , the centripetal force is given by force $\boldsymbol{F}(r)$:

$$m\frac{v^2}{r} = |\boldsymbol{F}(r)| = 6r^2,$$

Where v – is the speed of the body.

Thus

$$mv^2 = 6r^3 = 3U(r)$$

Therefore, the kinetic energy E_k is given by

$$E_k = \frac{mv^2}{2} = \frac{3U(r)}{2}$$

The total energy is given by the sum of the potential energy and kinetic energy:

$$E(r) = U(r) + E_k = U(r) + \frac{3}{2}U(r) = \frac{5}{2}U(r)$$

Thus

$$E(R) = \frac{5}{2}U(R) = \frac{5}{2}2(5^3)\text{ J} = 625\text{ J}$$

Answer: 625 J.