Answer on Question #56894, Physics / Mechanics | Relativity

Speedy Sue is going to an archery range. At the range she pulls the bowstring back by exerting 0.350m by exerting a fore that will increase from nothing to 250 N uniformly.

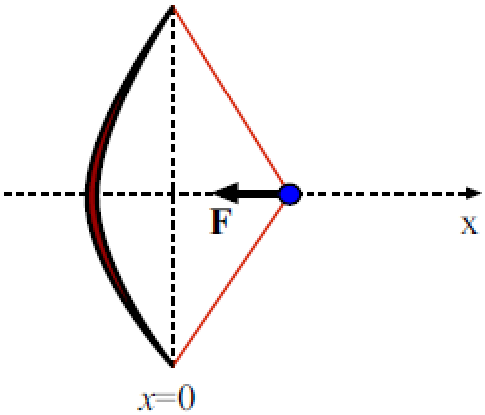
What is the spring constant of the bow?

How much work does Sue do in pulling the arrow back?

What is the speed of the arrow as it leaves the bow?

Solution:

While a bow string is not literally spring, it may behave like one in that it exerts a force on the thing attached to it (like a hand!) that is proportional to the distance of pull from the equilibrium position. The correspondence is illustrated in Fig.



We are told that when the string has been pulled back by 0.350m, the string exerts a restoring force of 250N. The magnitude of the string's force is equal to the spring constant k times the magnitude of the displacement; this gives us:

$$F = kx = 250 \text{ N}$$

Solving for k,

$$k = \frac{F}{x} = \frac{250}{0.350} = 714.29 \text{ N/m}$$

Still treating the bow string as if it were an ideal spring, we note that in pulling the string from a displacement of x = 0 to x = 0.400m the string does as amount of work on the hand given

$$W_{string} = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = 0 - \frac{1}{2} \cdot 714.29 \cdot 0.350^2 = -43.75 \text{ J}$$

Is this answer to the question? Not quite. . .we were really asked for the work done by the hand on the bow string. But at all times during the pulling, the hand exerted an equal and opposite force on the string. The force had the opposite direction, so the work that it did has the opposite sign. The work done (by the hand) in pulling the bow is W=+43.75 J.

The Conservation of Mechanical Energy give us that

$$K = W$$

$$\frac{mv^2}{2} = W = 43.75 \text{ J}$$

$$v = \sqrt{\frac{2W}{m}} = \sqrt{2 \cdot \frac{43.75}{m}} = \sqrt{\frac{87.5}{m}} \frac{\mathrm{m}}{\mathrm{s}}$$

where m is the arrow mass.

Answer: The (equivalent) spring constant of the bow is $k = 714.29\ \mathrm{N/m}$;

The work done in pulling the bow is 43.75 J.

The speed of the arrow depend on its mass: $v = \sqrt{\frac{87.5}{m}}\ \mathrm{m/s}$