Answer on Question #56598, Physics Mechanics Relativity

5.14 A stick of length l lies on horizontal table. It has a mass M and is free to move in any way on the table. A ball of mass m, moving perpendicularly to the stick at a distance d from its center with speed v collides elastically with it as shown in figure - 5.117. What quantities are conserved in the collision? What must be the mass of the ball so that it remains at rest immediately after collision.

Solution

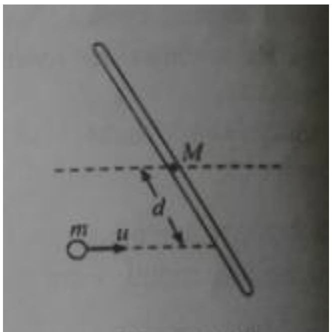

Figure 5.117

In the collision, linear momentum of the system (stick+ball), angularmomentum and kinetic energy are conserved. Using the law of conservation of angular momentum

$$mv = MV \quad (1)$$

By the law of conservation of angular momentum

$$mvd = J\omega \quad (2)$$

where $J = \frac{1}{12}Ml^2$

From the principle of conservation of kinetic energy

$$\frac{1}{2}mv^2=\frac{1}{2}J\omega^2+\frac{1}{2}MV^2 \qquad (3)$$

Then

$$mv^2=J\cdot\frac{m^2v^2d^2}{J^2}+M\frac{m^2v^2}{M^2}\Rightarrow m=\frac{m^2d^2}{\frac{1}{12}Ml^2}+\frac{m^2}{M}\Rightarrow$$
$$1=\frac{12d^2m}{Ml^2}+\frac{m}{M}\Rightarrow m=M\left(\frac{l^2}{l^2+12d^2}\right)$$

Answer: $m=M\left(\frac{l^2}{l^2+12d^2}\right)$.

5.15 A smooth uniform rod AB of mass M length l rotates freely with an angular velocity ω_0 in a horizontal plane about a stationary vertical axis passing through its end A. A small sleeve of mass m starts sliding along the rod from the point A. Find the velocity V of the sleeve relative to the rod at the moment it reaches the other end B.

Solution

We have used in non-inertial reference frame is rigidly connected to the rotating shaft. We draw the x-axis along AB. The origin coincides with the point A. The force of inertia

$$f_i=m\frac{dv_x}{dt}=m\omega^2x \qquad (1)$$

where m is the mass of sleeve.

Law of energy conservation

$$\frac{J\omega_0^2}{2} = \frac{J\omega^2}{2} + \frac{m(\omega x)^2}{2} + \frac{mv_x^2}{2} \qquad (2)$$

where $J = \frac{1}{3}Ml^2$ is the moment of inertia.

Then

$$\frac{Ml^2\omega_0^2}{6} = \frac{Ml^2\omega^2}{6} + \frac{m(x\omega)^2}{2} + \frac{mv_x^2}{2} \qquad (3)$$

From (3)

$$\omega^2 = \frac{1}{Ml^2 + 3mx^2}\left[Ml^2\omega_0^2 - 3mv_x^2\right] \qquad (4)$$

From (1) and (4)

$$m\frac{dv_x}{dt} = m\omega^2 x \Rightarrow \frac{dv_x}{dt} = \omega^2 x \Rightarrow \frac{dv_x}{dx}v_x$$

$$\frac{dv_x}{dx}v_x = \frac{1}{Ml^2 + 3mx^2}\left[Ml^2\omega_0^2 - 3mv_x^2\right]x \Rightarrow \frac{dv_x}{dx}\frac{v_x}{Ml^2\omega_0^2 - 3mv_x^2} = \frac{x}{Ml^2 + 3mx^2}dx$$

Then

$$\int_0^V \frac{dv_x}{dx}\frac{v_x}{Ml^2\omega_0^2 - 3mv_x^2} = \int_0^x \frac{x}{Ml^2 + 3mx^2}dx \Rightarrow$$

$$\frac{1}{6}\frac{\ln\left(\frac{Ml^2\omega_0^2}{Ml^2\omega_0^2 - 3mV^2}\right)}{m} = \frac{1}{6}\frac{\ln\left(\frac{Ml^2 + 3mx^2}{Ml^2}\right)}{m} \Rightarrow V(x) = \frac{\omega_0 x}{\sqrt{1 + \frac{3m}{M}\cdot\frac{x^2}{l^2}}}$$

$$V(x) = \frac{\omega_0 x}{\sqrt{1 + \frac{3m}{M}\cdot\frac{x^2}{l^2}}}$$

The velocity V of the sleeve relative to the rod at the moment it reaches the other end B

$$V(l)=\frac{\omega_0 l}{\sqrt{1+\frac{3m}{M}\cdot\frac{l^2}{l^2}}}=\frac{\omega_0 l}{\sqrt{1+\frac{3m}{M}}} \tag{5}$$

Answer: $V(l)=\frac{\omega_0 l}{\sqrt{1+\frac{3m}{M}}}$.