Answer on Question #56596-Physics-Mechanics-Relativity

5-3 A body of radius r and mass m is rolling horizontally without slipping with speed v. It then rolls up a hill to a maximum height h. If $h = 3\,v^2/4\,g$, what might the body be ? What is the body's moment of inertia.

Ans. $[R/\sqrt{2}\,]$

5-4 A conical pendulum is formed by a thin rod of length l and mass m, hinged at the upper end, rotates uniformly about a vertical axis passing through its upper end, with angular velocity ω. Find the angle θ between the rod and the vertical.

Ans. $[\theta = \cos^{-1}\left(\frac{3g}{2\omega^2 l}\right)]$

3.

Solution

The conservation of energy law:

$$\frac{I\omega^2}{2} + \frac{mv^2}{2} = mgh.$$

No slipping:

$$\omega = \frac{v}{r}.$$

The moment of inertia is

$$I = \frac{2mgh - mv^2}{\frac{v^2}{r^2}} = \frac{2mghr^2 - mv^2r^2}{v^2} = \frac{2mg\left(\frac{3v^2}{4g}\right)r^2 - mv^2r^2}{v^2} = \frac{1}{2}mr^2.$$

It is the ring of radius r.

4.

Solution

Looking at the rotating reference frame of the rod, there is a centrifugal force and gravity. Their torque must sum to zero because the rod is in equilibrium in that reference frame. For each bit of mass dm, we have

$$d\tau = (x\cos\theta)(\omega^2 x \sin\theta)dm.$$

plugging in $dm = \left(\frac{m}{l}\right)dx$ and simplify, I get

$$d\tau = \frac{m\omega^2 x^2}{l}\sin\theta\cos\theta\,dx.$$

$$\tau = \int_0^l \frac{m\omega^2 x^2}{l}\sin\theta\cos\theta\,dx = \frac{1}{3}m\omega^2 l^2 \sin\theta\cos\theta.$$

The torque provided by gravity is simply $mg\left(\frac{l}{2}\right)\sin\theta$. Equating with the centrifugal torque, I get

$$\frac{1}{3}m\omega^2 l^2 \sin\theta\cos\theta = mg\left(\frac{l}{2}\right)\sin\theta$$

$$\cos\theta = \frac{3g}{2\omega^2 l}$$

$$\theta = \cos^{-1}\left[\frac{3g}{2\omega^2 l}\right]$$