Answer on Question #56594, Physics Mechanics Relativity

5-19. The flywheel of a large motor in a factory has mass 30 kg and moment of inertia 67.5kg*m^2 about rotation axis. The motor develops a constant torque of 600Nt*m, and the flywheel starts from rest.

(a) What is the angular acceleration of the flywheel?

(b) What is its angular velocity after making 4 revolutions?

(c) How much work is done by the motor during the first 4 revolutions?

(d) What is the average power output of the motor during the first 4 revolutions?

Solution

(a)

$$M = J\varepsilon \quad (1)$$

where $M = 600 N \cdot m$ is a constant torque; ε is the angular acceleration of the flywheel; $J = 67.5 kg \cdot m^2$ is moment of inertia about rotation axis.

Then

$$\varepsilon = M / J = 600 N \cdot m / 67.5 kg \cdot m^2 \approx 8.89 rad / s^2$$

(b)

Angular velocity:

$$\omega = \omega_0 + \varepsilon t \quad (2)$$

$\omega_0 = 0$ (flywheel starts from rest).

The angle of rotation

$$\varphi = \varphi_0 + \omega_0 t + \frac{1}{2}\varepsilon t^2 \qquad (3)$$

where $\omega_0 = 0$ and $\varphi_0 = 0$ (flywheel starts from rest).

4 revolutions $\varphi = 4 \cdot 2\pi$, then from (3)

$$t = \sqrt{16\pi / \varepsilon} = 4\sqrt{\pi / \varepsilon} = 2.38s \qquad (4)$$

Angular velocity after making 4 revolutions:

$$\omega = \varepsilon t = \varepsilon 4\sqrt{\pi / \varepsilon} = 4\sqrt{\pi\varepsilon} = 4\sqrt{\pi \cdot 8.89} \approx 21.14 rad / s \qquad (5)$$

(c)

The work is done by the motor during the first 4 revolutions:

$$A = \frac{J\omega^2}{2} - \frac{J\omega_0^2}{2} = \frac{J\omega^2}{2} - 0 = \frac{J\omega^2}{2} = \frac{67.5 \cdot 8.89^2}{2} = 2667.33J = 2.67kJ \qquad (6)$$

(d)

The average power output of the motor during the first 4 revolutions:

$$\langle P \rangle = \frac{A}{t} = \frac{2667.33}{2.38} = 1120 Watt$$

5.20 Figure -5.118 shows three identical kiting spools at rest on a rough horizontal ground initially. In each case the string is pulled in the direction shown in figure. In each case it is given that spools rolls without slipping. In what direction will each spool will move and with what acceleration. Moment of inertia of each spool is spool is I and radius of inner tube is r and that of outer disc is R

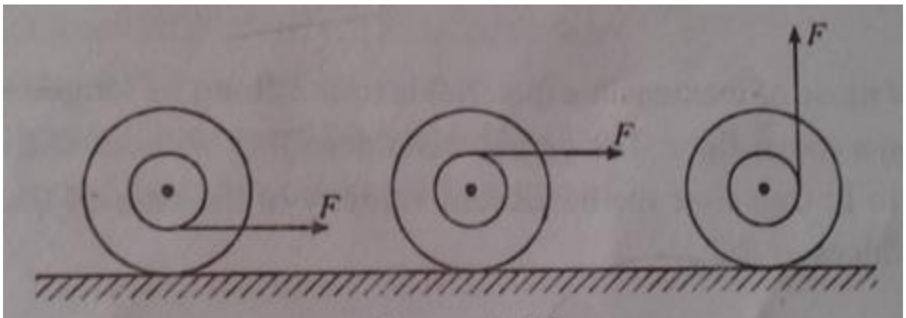

Figure 5.118

Answer:

It rolls to the right while rotating clockwise, if friction is sufficient to prevent it slipping on the table. The string winds up on the axle, so the spool moves to meet your hand as you pull on the string.

Now this gets more interesting. Raise the angle of string as you pull. At some critical angle, the spool will slide along the table without rotating. If you do this with a real spool, you will need to carefully maintain that constant angle as you pull. What is that critical angle?

If the angle is larger than the critical angle, the spool will roll in the opposite direction.

The free-body force diagram of the spool is easy to draw. The forces acting on it are (1) the gravitational force on the spool downward, (2) the upward normal force of the table on the spool (3) the force due to string tension, (4) the force due to friction at the table surface.

All of these are "unknowns". We assume we are dealing with a non-accelerating system, where linear motion and rotation are at constant speed. So we can use the fact that the net force is zero and the net torque is zero. By choosing a center of torques at the point of contact with the table, we eliminate the unknown weight, normal force, and force due to friction from the torque equation! We conclude that the torque due to the string, Tr is zero only when its line of action passes through the center of torques. Therefore the angle of the string with respect to the vertical is given by $\sin(\theta) = r/R$ where R is the outer radius of the spool and r is the axle radius. We might call this the formula for the spool sliding without rolling.

So far this has been a standard physics demonstration and problem. Here's a follow-up. Ask students to consider a solid cylinder with a mylar tape wrapped round it. The tape emerges at the table top and is pulled in a direction parallel to the table. Students try to apply the formula they just derived, and are stumped. The spool radius and the axle radius are identical, so application of the derived formula gives $\sin(\theta) = 1$ or $\theta = 90°$. We predict that when we pull the tape parallel to the table the spool will slide without rolling.