Answer on Question #56593, Physics Mechanics Relativity

5-17. A uniform rod of mass m and length l rests on a smooth horizontal surface. One of the ends of the rod is struck in a horizontal direction at right angles to the rod. As a result the rod obtains velocity v_0. Find the force with which one half of the rod will act the other in the process of motion.

Solution

Due to hitting of the ball the angular impulse by the rod about the $mv_0 l/2$. If ω is the angular velocity acquired by the rod. We have,

$$J\omega = \frac{mv_0 l}{2} \Rightarrow \frac{1}{12}ml^2\omega = \frac{mv_0 l}{2} \qquad (1)$$

or

$$\omega = \frac{6v_0}{l} \qquad (2)$$

For the frame of C.M., the rod is rotating about an axis passing through its mid point with the angular velocity ω. Hence the force exerted by the one half on the other= mass of half ·acceleration of the C.M. of that half in the C.M. frame

$$f = \frac{m}{2}\omega^2 \cdot \frac{l}{4} = \frac{m}{8}\omega^2 \cdot l = \frac{m}{8}\left(\frac{6v_0}{l}\right)^2 \cdot l = \frac{9mv_0^2}{2l}$$

Answer: $f = \frac{9mv_0^2}{2l}$

5-18. A smooth uniform rod AB of mass M length l rotates freely with an angular velocity ω_0 in a horizontal plane about a stationary vertical axis passing through its end A. A small sleeve of mass m starts sliding along the rod from the point A. Find the velocity V of the sleeve relative to the rod at the moment it reaches the other end B.

Solution

We have used in non-inertial reference frame is rigidly connected to the rotating shaft. We draw the x-axis along AB. The origin coincides with the point A. The force of inertia

$$f_i = m\frac{dv_x}{dt} = m\omega^2 x \tag{1}$$

where m is the mass of sleeve.

Law of energy conservation

$$\frac{J\omega_0^2}{2} = \frac{J\omega^2}{2} + \frac{m(\omega x)^2}{2} + \frac{mv_x^2}{2} \tag{2}$$

where $J = \frac{1}{3}Ml^2$ is the moment of inertia.

Then

$$\frac{Ml^2\omega_0^2}{6} = \frac{Ml^2\omega^2}{6} + \frac{m(x\omega)^2}{2} + \frac{mv_x^2}{2} \tag{3}$$

From (3)

$$\omega^2 = \frac{1}{Ml^2 + 3mx^2}\left[Ml^2\omega_0^2 - 3mv_x^2\right] \tag{4}$$

From (1) and (4)

$$m\frac{dv_x}{dt} = m\omega^2 x \Rightarrow \frac{dv_x}{dt} = \omega^2 x \Rightarrow \frac{dv_x}{dx} v_x$$

$$\frac{dv_x}{dx} v_x = \frac{1}{Ml^2 + 3mx^2}\left[Ml^2\omega_0^2 - 3mv_x^2\right] x \Rightarrow \frac{dv_x}{dx}\frac{v_x}{Ml^2\omega_0^2 - 3mv_x^2} = \frac{x}{Ml^2 + 3mx^2} dx$$

Then

$$\int_0^V \frac{dv_x}{dx}\frac{v_x}{Ml^2\omega_0^2 - 3mv_x^2} = \int_0^x \frac{x}{Ml^2 + 3mx^2} dx \Rightarrow$$

$$\frac{1}{6}\frac{\ln\left(\frac{Ml^2\omega_0^2}{Ml^2\omega_0^2 - 3mV^2}\right)}{m} = \frac{1}{6}\frac{\ln\left(\frac{Ml^2 + 3mx^2}{Ml^2}\right)}{m} \Rightarrow V(x) = \frac{\omega_0 x}{\sqrt{1 + \frac{3m}{M}\cdot\frac{x^2}{l^2}}}$$

$$V(x) = \frac{\omega_0 x}{\sqrt{1 + \frac{3m}{M}\cdot\frac{x^2}{l^2}}}$$

The velocity V of the sleeve relative to the rod at the moment it reaches the other end B

$$V(l) = \frac{\omega_0 l}{\sqrt{1 + \frac{3m}{M}\cdot\frac{l^2}{l^2}}} = \frac{\omega_0 l}{\sqrt{1 + \frac{3m}{M}}} \qquad (5)$$

Answer: $V(l) = \frac{\omega_0 l}{\sqrt{1 + \frac{3m}{M}}}$.