Answer on Question #56591, Physics Mechanics Relativity

A smooth uniform rod AB of mass M length l rotates freely with an angular velocity ω_0 in a horizontal plane about a stationary vertical axis passing through its end A. A small sleeve of mass m starts sliding along the rod from the point A. Find the velocity V of the sleeve relative to the rod at the moment it reaches the other end B.

Solution

We have used in non-inertial reference frame is rigidly connected to the rotating shaft. We draw the x-axis along AB. The origin coincides with the point A. The force of inertia

$$f_i = m\frac{dv_x}{dt} = m\omega^2 x \tag{1}$$

where m is the mass of sleeve.

Law of energy conservation

$$\frac{J\omega_0^2}{2} = \frac{J\omega^2}{2} + \frac{m(\omega x)^2}{2} + \frac{mv_x^2}{2} \tag{2}$$

where $J = \frac{1}{3}Ml^2$ is the moment of inertia.

Then

$$\frac{Ml^2\omega_0^2}{6}=\frac{Ml^2\omega^2}{6}+\frac{m\left(x\omega\right)^2}{2}+\frac{mv_x^2}{2} \tag{3}$$

From (3)

$$\omega^2=\frac{1}{Ml^2+3mx^2}\left[Ml^2\omega_0^2-3mv_x^2\right] \tag{4}$$

From (1) and (4)

$$m\frac{dv_x}{dt}=m\omega^2x\Rightarrow\frac{dv_x}{dt}=\omega^2x\Rightarrow\frac{dv_x}{dx}v_x$$
$$\frac{dv_x}{dx}v_x=\frac{1}{Ml^2+3mx^2}\left[Ml^2\omega_0^2-3mv_x^2\right]x\Rightarrow\frac{dv_x}{dx}\frac{v_x}{Ml^2\omega_0^2-3mv_x^2}=\frac{x}{Ml^2+3mx^2}dx$$

Then

$$\int_0^V\frac{dv_x}{dx}\frac{v_x}{Ml^2\omega_0^2-3mv_x^2}=\int_0^x\frac{x}{Ml^2+3mx^2}dx\Rightarrow$$
$$\frac{1}{6}\frac{\ln\left(\frac{Ml^2\omega_0^2}{Ml^2\omega_0^2-3mV^2}\right)}{m}=\frac{1}{6}\frac{\ln\left(\frac{Ml^2+3mx^2}{Ml^2}\right)}{m}\Rightarrow V\left(x\right)=\frac{\omega_0x}{\sqrt{1+\frac{3m}{M}\cdot\frac{x^2}{l^2}}}$$

$$V\left(x\right)=\frac{\omega_0x}{\sqrt{1+\frac{3m}{M}\cdot\frac{x^2}{l^2}}}$$

The velocity V of the sleeve relative to the rod at the moment it reaches the other end B

$$V\left(l\right)=\frac{\omega_0l}{\sqrt{1+\frac{3m}{M}\cdot\frac{l^2}{l^2}}}=\frac{\omega_0l}{\sqrt{1+\frac{3m}{M}}} \tag{5}$$

Answer: $V(l)=\dfrac{\omega_0 l}{\sqrt{1+\dfrac{3m}{M}}}$.