

Answer on Question #56589-Physics-Mechanics-Relativity

Question 6 has not seen.

5-5 A wheel of radius R rolls without slipping along the x axis with constant speed v_0 . Find the total distance covered by the point on the rim of the wheel during one complete revolution of the wheel.

Ans. $[8R]$

Solution

A cycloid is the curve traced by a point on the rim of a circular wheel as the wheel rolls along a straight line without slippage.

One arch of a cycloid (one complete revolution) generated by a circle of radius r can be parameterized by

$$x = r(t - \sin t); \quad y = r(1 - \cos t), \quad 0 \leq t \leq 2\pi$$

So,

$$\frac{dx}{dt} = r(1 - \cos t); \quad \frac{dy}{dt} = r \sin t$$

The arc length S of one arch is given by

$$\begin{aligned} S &= \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^{2\pi} r\sqrt{1 + \cos^2 t - 2\cos t + \sin^2 t} dt = \int_0^{2\pi} r\sqrt{2 - 2\cos t} dt \\ &= \int_0^{2\pi} 2r \sin \frac{t}{2} dt = 8r. \end{aligned}$$