Answer on Question #56588-Physics-Mechanics-Relativity

5-2 A ball of radius $R = 10$ cm rolls without slipping down an inclined plane so that its centre moves with constant acceleration 2.5 cm/s^2. After a time of 2 sec from the beginning of its motion, its position is as shown in figure-5.113. Find

(a) the velocities of point A, B and O.

(b) the acceleration of these points.

Ans. [10 cm/s, 7.1 cm/s, 0, 5.6 cm/s^2, 2.5 cm/s^2, 2.5 cm/s^2]

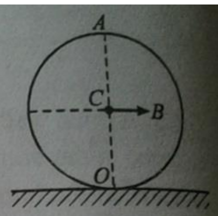

Figure 5.113

Solution

(a) No slipping:

$$\omega = \frac{v_{tr}}{R} = \frac{at}{R}.$$

For the points on the circumstance $v_{tr} = v_{rot} = \omega R$. For the center $v_{rot} = \omega \cdot 0 = 0$.

$$v_A = v_{tr} + v_{rot} = 2v_{tr} = 2at = 2 \cdot 2.5 \cdot 2 = 10\frac{cm}{s}.$$

$$v_B = \sqrt{v_{tr}^2 + v_{rot}^2} = \sqrt{2}v_{tr} = \sqrt{2}at = \sqrt{2} \cdot 2.5 \cdot 2 = 7.1\frac{cm}{s}.$$

$$v_O = v_{tr} - v_{rot} = 0.$$

(b) The acceleration is

$$\bar{a} = \overline{a_C} + \overline{a_\tau} + \overline{a_n}$$

$$a_C = a.$$

$$a_n = \frac{v_{tr}^2}{R} = \frac{(2.5 \cdot 2)^2}{10} = 2.5\frac{cm}{s^2} = a.$$

$$a_\tau = \frac{dv}{dt} = \frac{v_{tr}}{t} = \frac{at}{t} = a.$$

For A:

$$\overline{a_C} = \overline{a_\tau}.$$

$$a_A = \sqrt{a^2 + (2a)^2} = \sqrt{5}a = \sqrt{5} \cdot 2.5 = 5.6\frac{cm}{s^2}.$$

For B:

$$\overline{a_C} + \overline{a_n} = 0$$

$$a_B = a_\tau = a = 2.5\frac{cm}{s^2}.$$

For C:

$$\overline{a_C} + \overline{a_\tau} = 0$$

$$a_B = a_n = a = 2.5\frac{cm}{s^2}.$$

5-4 A conical pendulum is formed by a thin rod of length l and mass m, hinged at the upper end, rotates uniformly about a vertical axis passing through its upper end, with angular velocity ω. Find the angle θ between the rod and the vertical.

Ans. [$\theta = \cos^{-1}\left(\frac{3g}{2\omega^2 l}\right)$]

Solution

Looking at the rotating reference frame of the rod, there is a centrifugal force and gravity. Their torque must sum to zero because the rod is in equilibrium in that reference frame. For each bit of mass dm, we have

$$d\tau = (x\cos\theta)(\omega^2 x\sin\theta)dm.$$

plugging in $dm = \left(\frac{m}{l}\right)dx$ and simplify, I get

$$d\tau = \frac{m\omega^2 x^2}{l}\sin\theta\cos\theta\, dx.$$

$$\tau = \int_0^l \frac{m\omega^2 x^2}{l}\sin\theta\cos\theta\, dx = \frac{1}{3}m\omega^2 l^2\sin\theta\cos\theta.$$

The torque provided by gravity is simply $mg\left(\frac{l}{2}\right)\sin\theta$. Equating with the centrifugal torque, I get

$$\frac{1}{3}m\omega^2 l^2\sin\theta\cos\theta = mg\left(\frac{l}{2}\right)\sin\theta$$

$$\cos\theta = \frac{3g}{2\omega^2 l}$$

$$\theta = \cos^{-1}\left[\frac{3g}{2\omega^2 l}\right]$$