Answer on Question#56498 - Physics - Other

4/ A $m = 28\text{kg}$ rock approaches the foot of a hill with a speed of $v_i = 15\frac{\text{m}}{\text{s}}$. This hill slopes upward at a constant angle of $\theta = 40.0°$ above the horizontal. The coefficients of static and kinetic friction between the hill and the rock are $\mu_s = 0.75$ and $\mu_k = 0.20$, respectively.

(a) Use energy conservation to find the maximum height above the foot of the hill reached by the rock.

(b) Will the rock remain at rest at its highest point, or will it slide back down the hill?

(c) If the rock does slide back down, find its speed when it returns to the bottom of the hill.

Solution:

(a) Let the maximum height be h, then the distance along the slope from the foot to this point is

$$l = \frac{h}{\sin\theta}$$

The projection of gravitational force on the inclined surface is

$$F_{\parallel} = mg\sin\theta$$

The ground reaction force is

$$F_{\perp} = mg\cos\theta$$

The force of kinetic friction is

$$F_k = F_{\perp}\mu_k = mg\mu_k\cos\theta$$

The work done by the force of kinetic friction is

$$W_k = F_k \cdot l = mgl\mu_k\cos\theta$$

According to the law of conservation of energy

$$\frac{mv_i^2}{2} = mgh + W_k$$

Therefore

$$\frac{mv_i^2}{2} = mgh + mg\frac{h}{\sin\theta}\mu_k\cos\theta = mgh(1 + \mu_k\cot\theta)$$

$$h = \frac{v_i^2}{2g(1 + \mu_k\cot\theta)} = \frac{\left(15\frac{\text{m}}{\text{s}}\right)^2}{2\cdot 9.8\frac{\text{m}}{\text{s}^2}(1 + 0.2\cot 40°)} = 9.27\text{m}$$

(b) If the force of the static friction F_s is greater then $F_{\parallel}$, then the rock won't slide down. The force of static friction is given by

$$F_s = F_{\perp}\mu_s = mg\mu_s\cos\theta$$

Thus

$$\frac{F_s}{F_{\parallel}} = \frac{mg\mu_s\cos\theta}{mg\sin\theta} = \frac{\mu_s}{\tan\theta} = \frac{0.75}{\tan 40°} = 0.89$$

$$F_s < F_{\parallel}$$

It means that the rock will slide down.

(c) The work done by the force of kinetic friction is the same as in part (a):

$$W_k = F_k \cdot l = mgh\mu_k \cot\theta$$

According to the law of conservation of energy

$$mgh = \frac{mv_f^2}{2} + W_k,$$

Where v_f – is the speed of the rock at the bottom of the slope.

Thus

$$mgh = \frac{mv_f^2}{2} + mgh\mu_k \cot\theta$$

$$v_f = \sqrt{2gh(1 - \mu_k \cot\theta)} = \sqrt{2 \cdot 9.8\frac{\text{m}}{\text{s}^2} \cdot 9.27\text{m}(1 - 0.2 \cdot \cot 40°)} = 11.76\frac{\text{m}}{\text{s}}$$

Answer:

(a) 9.27m

(b) Will slide down

(c) $11.76\frac{\text{m}}{\text{s}}$