Answer on Question#56496 - Physics - Other

1/ (20 pts) To stretch a spring 3.00 cm from its unstretched length, 12.0 J of work must be done.

(a) What is the force constant of this spring?

(b) What magnitude force is needed to stretch the spring 3.00 cm from its unstretched length?

(c) How much work must be done to compress this spring 4.00 cm from its unstretched length and what force is needed to stretch it this distance?

2/ (20 pts) A particle with mass m is acted on by a conservative force and moves along a path given by x = Acosωt and y = Bsinωt, where A, and B are constants.

(a) Find the components of the force that acts on the particle.

(b) Find the potential energy of the particle as a function of x and y. Take U = 0 when x = 0 and y = 0.

(c) Find the total energy of the particle when x = A.

<u>Solution:</u>

1. The potential energy of the stretched spring with force constant k is given by
$$E_p = \frac{kx^2}{2},$$
Where x – is the distance it stretched from the unstretched length.
Since $x = 0.03\mathrm{m}$ and $E_p = 12\mathrm{J}$, we obtain
$$k = \frac{2E_p}{x^2} = \frac{2 \cdot 12\mathrm{J}}{(0.03\mathrm{m})^2} = 26.7\frac{\mathrm{kN}}{\mathrm{m}}$$
The force needed to stretch this spring $x = 3.00$ cm from its unstretched length is
$$F = kx = 26.7\frac{\mathrm{kN}}{\mathrm{m}} \cdot 0.03\mathrm{m} = 800\mathrm{N}$$
The work to compress this spring $l = 4.00$ cm from its unstretched length is
$$E_{comp} = \frac{kl^2}{2} = \frac{26.7\frac{\mathrm{kN}}{\mathrm{m}} \cdot (0.04\mathrm{m})^2}{2} = 21.3\mathrm{J}$$
And the force needed to hold the spring in this position is
$$F_{comp} = kl = 26.7\frac{\mathrm{kN}}{\mathrm{m}} \cdot 0.04\mathrm{m} = 1067\mathrm{N}$$
2. According to the Newton's second law
$$\boldsymbol{F} = m\boldsymbol{a}$$
Therefore
$$F_x = m\frac{d^2x}{dt^2} = -mA\omega^2 \cos\omega t = -m\omega^2 x$$
$$F_y = m\frac{d^2y}{dt^2} = -mB\omega^2 \sin\omega t = -m\omega^2 y$$
It's easy to see that the potential energy U is given by

$$U = \frac{m\omega^2(x^2 + y^2)}{2}$$

It meets the requirement $U(0,0) = 0$.

Indeed

$$F_x = -\frac{\partial U}{\partial x} = -m\omega^2 x$$

$$F_y = -\frac{\partial U}{\partial y} = -m\omega^2 y$$

The total energy of the particle is given by (sum of potential and kinetic energies)

$$W = U + \frac{m(\dot{x}^2 + \dot{y}^2)}{2} = \frac{m\omega^2(x^2 + y^2)}{2} + \frac{m(\dot{x}^2 + \dot{y}^2)}{2}$$

$x = A$:

$$\cos\omega t = 1$$

Thus

$$\sin\omega t = 0$$

And

$$\dot{x} = -A\omega\sin\omega t = 0$$

$$\dot{y} = B\omega\cos\omega t = B\omega$$

$$y = B\sin\omega t = 0$$

The total energy then

$$W = \frac{m\omega^2(A^2 + 0^2)}{2} + \frac{m(0^2 + (B\omega)^2)}{2} = \frac{m\omega^2(A^2 + B^2)}{2}$$

<u>Answer:</u>

1.
   (a) $26.7\frac{\mathrm{kN}}{\mathrm{m}}$
   (b) $800\mathrm{N}$
   (c) $21.3\mathrm{J}$, $1067\mathrm{N}$
2.
   (a) $F_x = -mA\omega^2\cos\omega t = -m\omega^2 x$
   $F_y = -mB\omega^2\sin\omega t = -m\omega^2 y$
   (b) $\frac{m\omega^2(x^2+y^2)}{2}$
   (c) $\frac{m\omega^2(A^2+B^2)}{2}$