Answer on Question 56222, Physics, Mechanics, Relativity

Question:

A synchronous satellite circles the Earth eastward above equator once every $24h$ and stays over the same spot on the Earth because the Earth is rotating at the same rate. What is the velocity of the synchronous satellite?

a) $9800\ m/s$

b) $4300\ m/s$

c) $3100\ m/s$

d) $2400\ m/s$

Solution:

Let's first find the orbital radius of the synchronous satellite. When the satellite orbits the Earth the centripetal force acts on it:

$$F_c = \frac{m_{sat} v^2}{R_{sat}},$$

where, m_{sat} is the mass of the satellite, v is the orbital speed of the satellite and R_{sat} is the orbital radius of the satellite.

From the other hand, the gravitational force attracts the satellite towards the Earth, and we can write:

$$F_{grav} = G \frac{m_{sat} M_E}{R_{sat}^2},$$

where, $G = 6.67 \cdot 10^{-11} \frac{Nm^2}{kg^2}$ is the gravitational constant, $M_E = 5.98 \cdot 10^{24} kg$ is the mass of the Earth.

Since, $F_c = F_{grav}$, we obtain:

$$\frac{v^2}{R_{sat}} = G \frac{M_E}{R_{sat}^2}.$$

Because the satellite travels around the entire circumference of the circle which is $2\pi R_{sat}$ in the period T, this means that the orbital speed must be $v = \frac{2\pi R_{sat}}{T}$. Substituting the expression for the orbital speed into the last equation we get:

$$\frac{\left(\frac{2\pi R_{sat}}{T}\right)^2}{R_{sat}} = G\frac{M_E}{R_{sat}^2}.$$

Finally, after simplification we get the formula for the orbital radius of the synchronous satellite:

$$R_{sat} = \sqrt[3]{\frac{GM_E T^2}{4\pi^2}} = \sqrt[3]{\frac{6.67 \cdot 10^{-11}\frac{Nm^2}{kg^2} \cdot 5.98 \cdot 10^{24} kg \cdot (24 \cdot 3600 s)^2}{4 \cdot (3.14)^2}}$$
$$= 4.226 \cdot 10^7 m.$$

Then, we can find the orbital speed of the synchronous satellite:

$$v = \frac{2\pi R_{sat}}{T} = \frac{2 \cdot 3.14 \cdot 4.226 \cdot 10^7 m}{24 \cdot 3600 s} = 3072\frac{m}{s} \sim 3100\frac{m}{s}.$$

Answer:

c) $3100\ m/s$